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**Issues in Operations Management and Marketing Interface Research: Competition, Product Line Design, and Channel Coordination**

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# Issues in Operations Management and Marketing Interface Research: Competition, Product Line Design, and Channel Coordination

**by**

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To my wife Ling, my son Ethan, and my parents for their love and support.

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**Issues in Operations Management and Marketing Interface Research: Competition, Product Line Design, and Channel Coordination**

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This dissertation studies important issues in supply chain management and marketing interface research: competition, product line design, and channel efficiency, at the presence of vertically differentiated products. Vertical differentiation as a means of price discrimination has been well-studied in both economics and marketing literature. However, less attention has been paid on how vertical differentiation has been operationalized. In this dissertation, we focus our study on two types of vertical differentiation: the one created by a product line which is produced by the same firm, and the one created by products from different firms. We especially are interested in the so-called private label products vs. the national brand products. Specifically, this dissertation explores how vertical differentiation can affect the interactions among the members of a supply chain in several different contexts. In the first piece of

work, we use a game theoretic model to explore how the ability of a retailer to introduce a private label product affects its interaction with a manufacturer of a national brand. In the second essay, we are investigating how an original equipment manufacturer (OEM) will be affected by the entry of a competitor when there are strategic suppliers of a critical component. If these suppliers behave strategically, it is not clear that the entry of other players will necessarily be harmful to the incumbent. In the last work, we pay our attention to an emerging change happening in the industry: some retailers begin to sell their private labels through their competitors. We investigate the strategic role of a retailer selling her own private label products through another retailer. In summary, this dissertation illustrates how vertical differentiation play a crucial role in firms' supply chain as well as marketing strategies. Therefore, it is important for firms to recognize these strategic issues related to vertically differentiated products while making operations/marketing decisions.

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# Chapter 1

# Executive Summary

Operations management (OM) and marketing as stand-alone disciplines have been deeply pursued by both academia and practitioners during the past decades. Operations Management discipline has emerged from fields of operations research and industrial engineering and has become a major focus of business research, together with finance and marketing. During the last forty years or so, it has gone through different emphases on topics studied, such as MRP, JIT, and TQM. Starting from the late 1990s, researchers have recognized that operations is only one functional area and, to be successful, operations management researchers must interface with their peers in the research fields of marketing, finance, engineering, and other functional areas. Therefore, in addition to the knowledge of operations research tools, operations management researchers must also understand business strategy, marketing concepts, financial tools, and effectiveness of information technology (systems). In this dissertation, we focus our research on interfaces between operations management and marketing.

As the business environment becomes more competitive, supply chain management has become a topic of interest to many people. Efficient and effective supply chain management strategy has been widely recognized as value maximization, process integration, responsiveness improvement, and cycle time reduction. Although much attention has been paid to the increase in efficiency that has occurred in retail channels over the past twenty years, the gains are typically attributed to either the role of information technology in facilitating practices such as vendor managed inventory (VMI), collaborative planning forecasting and replenishment (CPFR), etc. or to the consolidation among retailers and redesign of store formats, i.e. e-tailing and big-box. One of the consequences of having fewer, bigger retailers, is that many of them now have sufficient economies of scale to be able to produce their own private label products. For example, H-E-B, which operates over 300 grocery stores in Texas and Mexico, carries more than 3,000 items under the brand names of Hill Country Fare and H-E-B, and operates its own manufacturing operations for many of these. Once a retailer has developed the capability of producing its own private label in a category, it has one more alternative to consider in its assortment, pricing and promotional planning, and this may give it more leverage with respect to a national brand manufacturer(s). On the other hand, because the retailer is self-interested, her decision to develop private label capability may not necessarily benefit the supply chain as a whole.

In the first essay, we use a game theoretical model to explore how the ability of a retailer to introduce a private label product affects its interactions with a manufacturer of a national brand. This work was motivated by the observation that although a retailer's decision to introduce a private label product may lead to a sub-optimal product line, it will also tend to put pressure on the manufacturer's margin, dampening the effects of double-marginalization. The main thrust of this work is to explore this trade-off between the efficiency of the product line and double marginalization, and we pay special attention to how this trade-off may depend on the characteristics of the product category.

We first distinguish between the characteristics of private label products that are *structurally efficient*, i.e. they would be introduced as product-line extensions by a vertically integrated channel, vs. those that are *structurally inefficient*. Then we show that although retailers may introduce private labels that are structurally inefficient, depending upon the cost structure, this may or may not benefit the overall channel of distribution. When the development cost of the private label is not too high, its mitigating effects upon double marginalization can dominate its structural inefficiency. In an extension to our basic model, we consider the strategic interaction between a retailer's ability to develop a private label and her ability to stimulate demand through promotion. Interestingly, the two capabilities can be either strategic complements or substitutes, depending upon the efficiency (cost/quality ratio) of the private label relative to the national brand.

In the second essay, we investigate how an original equipment manufacturer (OEM) will be affected by the entry of a competitor when there are strategic suppliers of a critical component. For example, Apple cannot produce the iPod without high quality compact hard drives, of which there are relatively few suppliers. If these suppliers behave strategically, it is not clear that the entry of other personal music devices will necessarily be harmful to Apple. In our analysis, we show that, if the relative perceived quality of the entrant's product is neither too low nor too high, its entry may benefit the incumbent. The reason is that, in response to the entry of the lower quality OEM, the suppliers respond by increasing their output quantities which would decrease the price at which the components can be acquired. As a consequence, it is possible for the incumbent OEM to benefit more from the reduction in procurement cost than he is hurt by the cannibalization from the entrant OEM. Of course, in order for this to happen, the supply industry must be sufficiently strategic. That is, if there are too many suppliers, then the entry of the new OEM does not have a sufficient impact on the price of components to provide enough benefit to the incumbent OEM to offset the cannibalization.

The above result is noteworthy because, while others have demonstrated that a firm can benefit from competition from a lower quality product, the existing results depend upon network effects in one form or another. Our result is independent of any network effects, and is instead driven by the strategic behavior of suppliers.

Both essays demonstrate the impact of supply chain structure on supply chain efficiency as well as individual supply chain members. In the last essay, we turn our attention to the supply chain design issue from a different angle. During the last decades, private label products are widely seen in retailscape. The world is changing from dominated by manufacturer brands to a mix of manufacturer brands and retailer owned brands. The name "private labels" comes from the fact that most of the retailer brands, if not all, were carried exclusively by the owners of those brands. For example, major U.S. retail chains such as Wal-Mart, Target, JCPenney, and big-boxes such as Costco and Sam's Club, have aggressively entered the private label markets during the last decades. Costco's Kirkland Signature, JCPenney's Arizona, and other retailer owned brands become more and more popular among consumers now. However, over time, we started to notice a change in the retail industry. A few retailers begin to distribute their private label products through their competing retailers. For example, starting on the fall of 2008, Safeway began to roll out its popular O's organic foods and Eating Right healthy foods store brands to a wider audience – competing food retailers in the U.S. – along with to grocers globally. At the end of year 2008, as one of the largest office supply providers in U.S., OfficeMax partnered with Safeway to provide office products and school supplies to grocery stores. More recently, Sears Holdings Corp. has agreed to sell its popular Craftsman tool brand through Ace Hardware stores, as the company turns again to outsiders to help grow its sales. We are interested in the following research questions. First, if you are a marketing manager of Safeway or OfficeMax, when should you keep your retailer brands private? When should you share your retailer brands with your competing retailers? If you are the marketing manager of the national brand manufacturer, what's the implication for you when your retailer keeps its retailer brand private or shares it with other retailers of yours? Finally, is it to the channel's best interest when a retailer shares its retailer brand with its competitors?

We study the problem in a supply chain with a national brand manufacturer selling to through two retailers. One of the retailers owns a retailer brand. We first examine the basic case in which retailers operate in independent markets. Our analysis shows that selling the retailer brand through the other retailer has a strategic effect of inducing a lower wholesale price of the national brand. We also find that the total sales of retailer brand may decrease if the retailer sells her retailer brand to the other retailer. However, according to our analysis, we are able to show that it may be at the retailer's best interest to share her retailer brand with the competitor when the perceived quality level of the retailer brand is neither too low nor too high and when the market size of the other retailer is large enough. By exploring the case in which retailers compete in the same market, we find that competition plays an important role in deciding whether to sell the retailer brand through the competitor.

The rest of the dissertation is organized as follows. In Chapter 2, we discuss the basic modeling components which are common adopted in each of the three essays. In Chapter 3, we discuss the impact of private label development on supply chain efficiency. In Chapter 4, we discuss the strategic effects of competition and product line efficiency in the presence of strategic suppliers. In Chapter 5, we investigate the underlying reasoning of why retailers choose to sell their retailer brands through competitors. Finally, in Chapter 6, we provide managerial implications for firms involving vertically differentiated products and point to directions for future research.

# Chapter 2

# Model Components

## 2.1 The Product Space

Products in today's markets are almost always differentiated by some characteristic.[1] The question is, how can we describe the differentiation between the products within a category. This question has been answered by several economists. Among others, Hotelling (1929, [20]), Chamberlin (1951, [7]; 1962, [8]), and Lancaster (1966, [25]) provide good answers. A product can be described as a bundle of characteristics: quality, location, time, availability, consumers' information about its existence and quality, and so on. Each consumer has a ranking over the mix of variables.

We may include all potential characteristics for a product. By doing this, we provide a rich description about the product. However, it is likely to be of little help in studying issues of supply chain management. Both in empirical work and theoretical work, researchers focus their attention on a small subset of characteristics and on a special (but, if possible, reasonable) description of preferences. There are three commonly adopted approaches in the literature.

[1] The main content of this section draws mainly from Tirole (1988, [47]).

### 2.1.1 Vertical Differentiation

In a vertically differentiated product category, all consumers agree over the most preferred mix of characteristics and, more generally, over the preference ordering. A typical example is quality. Most agree that higher quality is preferable – for instance, that a Volve is preferable to a Hyundai. This preference does not imply that all consumers buy the Volve. However, more consumers may still purchase the latter. The consumers' income and the prices of the cars, and of servicing them, determine the consumers' ultimate choice. We will describe this point in detail in the following sections. Similarly, a smaller and more powerful computer is preferable to a larger, less powerful one. At equal prices there is a natural ordering over the characteristic space.

In our work, we focus exclusively in vertically differentiated product categories. We deter the detailed discussion to the following sections.

### 2.1.2 Horizontal Differentiation

For some characteristics, the optimal choice (at equal prices) depends on the particular consumer. Tastes vary in the population. An obvious example is the case of colors. Another example is location. The University of Texas at Austin students are likely to prefer textbooks that are available in Austin to textbooks that are physically the same but are available only in Paris. Similarly, consumers will prefer to go to a store or supermarket that is near their places. Different from the case of vertical differentiation, in such cases of horizontal or "spatial" differentiation, there are no "goods" or "bads".

**Example**

A simple example, the so-called "linear city" model, is provided in Hotelling (1929, [20]). Consider a "linear city" of length 1. Consumers are distributed uniformly along the city. Two shops, located at the two ends of the city, both sell the same physical product. The location of shop 1 is $x = 0$, and that of shop 2 is $x = 1$. (See Figure 2.1.) Consumers have transportation cost $t$ per unit of length. They consume one or zero unit of the product. Let $p_1$ and $p_2$ denote the prices charged by the two shops. The "generalized price" of going to shop 1 (respectively, shop 2) for a consumer with coordinate $x$ is $p_1 + tx$ (respectively, $p_2 + t(1 - x)$). If $\bar{s}$ denotes the surplus enjoyed by each consumer when he is consuming the product, the utility of a consumer located at $x$ is

$$\bar{s} - p_1 - tx \tag{2.1}$$

if he buys from shop 1,

$$\bar{s} - p_2 - t(1 - x) \tag{2.2}$$

if he buys from shop 2, and zero otherwise. The demand functions can be derived from the above utility functions. We omit the details in this dissertation. Interested readers can refer to Tirole (1988, [47]) for detailed derivation.

### 2.1.3 "Products-Characteristics" Approach

Products are defined as bundles of characteristics, and consumers have preferences over characteristics. The consumers may have heterogeneous preferences over characteristics. We assume that each consumer only consumes one unit of the products under consideration. We may also assume that consumers can consume multiple products. Furthermore, we can assume that what the consumers care about are the characteristics of the products. For examples, there are multiple products contain protein and vitamins, which consumers care about from those products. Each product contains a combination of protein and vitamins. When consumers consume a bundle of products, they are indifferent among bundles providing the same amount of protein and vitamins. This approach is pioneered by Lancaster (1966, [25]).

## 2.2 The Demand Model

In our work, we adopt the vertical differentiation approach as described in Section 2.1.1. In this section, we derive the demand model which will be used in the rest of this dissertation based on the vertical differentiation approach and consumer utility theory from economics.

Each consumer consumes one or zero units of a product. The product is characterized by a quality index $q$. When there are several qualities in a product category, we will often talk about these different qualities as being "different products." For the moment, let's focus our attention to the case of a single quality/product.

A consumer has the following preferences:

$$U = \begin{cases} \theta q - p & \text{if he buys a product with quality } q \text{ at price } p, \\ 0 & \text{if he does not buy.} \end{cases} \tag{2.3}$$

$U$ should be thought of as the utility derived from the consumption of the product. $q$ is a positive real number that describes the quality of the product. The utility is separable in quality and price. $\theta$, a positive real number, is a taste parameter, or a valuation for quality parameter by another name. All consumers prefer high quality, for a given price; however, a consumer with a high $\theta$ is more willing to pay to purchase high quality. We assume that the taste parameter $\theta$ is continuously distributed with PDF, $f(\theta)$, and CDF, $F(\theta)$, over the interval [0, 1], where $F(0) = 0$ and $F(1) = 1$. Thus, $F(\theta)$ is the fraction of consumers with a taste parameter of less than $\theta$.

Alternatively, we can also interpret $\theta$ as the inverse of the marginal rate of substitution between income and quality rather than as a taste parameter. The consumer's preferences as described in equation (2.3) can be rearranged as

$$U = \begin{cases} q - (1/\theta)\, p & \text{if he buys a product with quality } q \text{ at price } p, \\ 0 & \text{if he does not buy.} \end{cases} \tag{2.4}$$

On this interpretation, all consumers derive the same utility from the product, but they have different incomes and, therefore, different marginal rates of substitution between income and quality $(1/\theta)$. Wealthier consumers have a lower "marginal utility of income" or, equivalently, a higher $\theta$.

The demand function for the product based on this particular utility function can be derived as follows. If there is only one product available in the market, the demand for the product is equal to the number of consumers who purchase the product. That is, the number of consumers with valuation

$\theta$ such that $\theta q \geq p$. In other words, the demand for the product is

$$D(p) = N\left[1 - F(p/q)\right], \tag{2.5}$$

where $N$ is the total number of consumers. In this dissertation, we normalize the total number of consumers to be $N = 1$.

If there are several qualities offered in the market, the consumers choose among these qualities as well as choosing whether to buy at all. We assume that all consumers have unit demands – i.e., they consume at most one unit of the product – whatever the quality. Particularly, we consider a product category with two different qualities. We may consider product categories with more than two qualities. However, for the ease of exposition, we focus our attention in this dissertation to categories with two products with qualities $q_1 < q_2$, which are sold at prices $p_1 < p_2$.

Consumers with valuation $\theta$ will obtain a net utility of $\theta q_i - p_i$ from purchasing product $i$, $i = 1, 2$, or 0 from buying nothing. Each consumer will decide to purchase either product 1, or product 2, or nothing, depending upon which of these options maximizes his or her net utility. For any $0 \leq p_1 < p_2$, the market segmentation can be determined from the individual rationality and incentive compatibility constraints. Let $\theta_i = p_i/q_i$, $i = 1, 2$. Any consumer with valuation $\theta \geq \theta_2$ prefers buying product 2 to not buying at all. Similarly, any consumers with valuation $\theta \geq \theta_1$ prefers buying product 1 to not buying at all. Define $\theta_{1,2} = \frac{p_2 - p_1}{q_2 - q_1}$. Consumers with valuation $\theta \geq \theta_{1,2}$ prefer buying product 2 to product 1:

$$\begin{aligned}
& (\theta q_2 - p_2) - (\theta q_1 - p_1) \\
= \; & (q_2 - q_1)\left(\theta - \frac{p_2 - p_1}{q_2 - q_1}\right) \\
\geq \; & 0,
\end{aligned}$$

when $\theta \geq \theta_{1,2}$.

We have the following properties regarding the three values $\theta_1$, $\theta_2$, and $\theta_{1,2}$.

**Lemma 2.2.1.** *Let* $\theta_1 = \frac{p_1}{q_1}$, $\theta_2 = \frac{p_2}{q_2}$, *and* $\theta_{1,2} = \frac{p_2 - p_1}{q_2 - q_1}$. *We have:*

1. *If* $\theta_1 \leq \theta_2$ *then* $\theta_1 \leq \theta_2 \leq \theta_{1,2}$*;*

2. *If* $\theta_2 \leq \theta_1$ *then* $\theta_{1,2} \leq \theta_2 \leq \theta_1$*.*

*Proof.* All proofs are provided in the appendix for the corresponding chapters.

□

As a consequence of these relationships, the possible values for $p_1$ and $p_2$ can be divided into three mutually exclusive and collectively exhaustive regions $R_1$, $R_2$, and $R_{1,2}$ (the subscript "1" denotes product 1, "2" denotes product 2, and "$1,2$" denotes both products) as follows:

$$
\begin{aligned}
R_1 &\equiv \{(p_1,\, p_2):\ p_1 \le p_2 + q_1 - q_2\}\,, \\
R_2 &\equiv \left\{(p_1,\, p_2):\ p_1 \ge \frac{q_1}{q_2} p_2\right\}, \\
R_{1,2} &\equiv \left\{(p_1,\, p_2):\ p_2 + q_1 - q_2 < p_1 < \frac{q_1}{q_2} p_2\right\}.
\end{aligned}
\tag{2.6}
$$

Define $Q_1\,(p_1,\, p_2)$ and $Q_2\,(p_1,\, p_2)$ to be the quantities of product 1 and 2 that are sold when the retail prices are $p_1$ and $p_2$. The following result then follows from applying the standard approach of identifying the valuations of marginal consumers:

**Lemma 2.2.2.** *For any prices $(p_1, p_2) \in R_1 \cup R_2 \cup R_{1,2}$, consumer purchasing behavior can be characterized as follows:*

1. *If $(p_1, p_2) \in R_1$, only product 1 experiences positive demand, and $Q_1\,(p_1, p_2) = 1 - F\,(\theta_1)$.*

2. *If $(p_1, p_2) \in R_{1,2}$, both products experience positive demand, and $Q_1\,(p_1, p_2) = F\,(\theta_{1,2}) - F\,(\theta_1)$, and $Q_2\,(p_1, p_2) = 1 - F\,(\theta_{1,2})$.*

3. *If $(p_1, p_2) \in R_2$, only product 2 experiences positive demand, and $Q_2\,(p_1, p_2) = 1 - F\,(\theta_2)$.*

## 2.3 Distribution of Consumer Valuation

In order to facilitate the analysis of the models under consideration, we assume that consumer valuations follow a family $\mathcal{F}^K$ of distributions on the interval [0, 1], where $K \in (0, \infty)$. The CDF and PDF of the distribution family are characterized by the single parameter $K > 0$ as follows:

$$F(\theta;\, K) = 1 - (1-\theta)^K \;\; and \;\; f(\theta;\, K) = K\,(1-\theta)^{K-1}\,, \tag{2.7}$$

where, $0 \leq \theta \leq 1$. It is easy to confirm that $\mathcal{F}^K$ includes the uniform distribution ($K = 1$). Thus, our distributional assumption is more general than most other analytical models of vertically differentiated products, which nearly universally rely upon the uniform distribution. To our knowledge, $\mathcal{F}^K$ was first applied to market segmentation by Debo et al. (2005, [13]), but it has not been specifically used to study the interaction between vertically differentiated products. As shown in Figure 2.2, which plots the density $f(\theta;\, K)$ and distribution function $F(\theta;\, K)$ for three different values of $K$, $\mathcal{F}^K$ allows us to consider situations in which the concentration of mass can be either at the low or the high end of the spectrum. In the figure it is easy to see that: when $K < 1$, consumer valuations are concentrated at the high end; when $K > 1$, they are concentrated at the low end; and when $K = 1$, they are uniformly distributed.

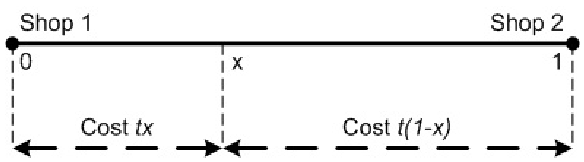
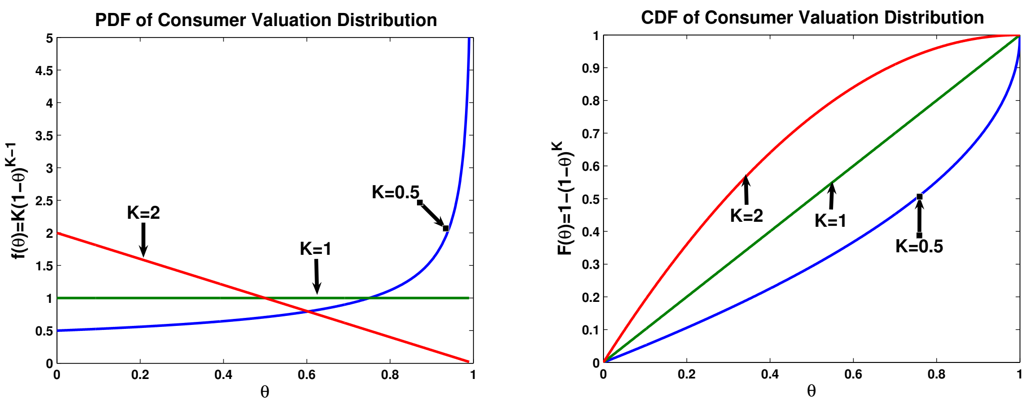



Figure 2.1: The linear city.




(a) PDF

(b) CDF

Figure 2.2: (a) $f(\theta) = K(1-\theta)^{K-1}$; (b) $F(\theta) = 1-(1-\theta)^K$ for $K = 0.5$, $K = 1$, and $K = 2$.

# Chapter 3

# Private Label Development

## 3.1 Introduction

Although much attention has been paid to the increase in efficiency that has occurred in retail channels over the past twenty years, the gains are typically attributed to either the role of information technology in facilitating practices such as vendor managed inventory (VMI), collaborative planning forecasting and replenishment (CPFR), etc. or to the consolidation among retailers and redesign of store formats, i.e. e-tailing and big-box. One of the consequences of having fewer, bigger retailers, is that many of them now have sufficient economies of scale to be able to produce their own private label products. For example, H-E-B, which operates over 300 grocery stores in Texas and Mexico, carries more than 3,000 items under the brand names of Hill Country Fare and H-E-B, and operates its own manufacturing operations for many of these.[1] Once a retailer has developed the capability of producing its own private label in a category, it has one more alternative to consider in its assortment, pricing and promotional planning, and this may give it more leverage with respect to a national brand manufacturer(s). On the other

[1] H-E-B operates the largest dairy and bakery production facilities in the state of Texas (http://www.heb.com/aboutHEB/history.jsp).

hand, because the retailer is self-interested, her decision to develop private label capability may not necessarily benefit the supply chain as a whole.

In this chapter, we investigate how a retailer's ability to develop private label capability affects the efficiency of a decentralized supply chain consisting of a single manufacturer of a national brand and a single retailer. To the extent that private labels are often perceived to be of lower quality than the national brands, a retailers development of private label capability is effectively a *product line extension*. However, the fact that the private label affects the strategic interactions with the national brand manufacturer is a critical distinction between this product line decision and the traditional one that has been studied by Mussa and Rosen (1978, [34]), Moorthy (1984, [31]), Moorthy and Png (1992, [32]), Desai (2001, [16]), among others, where the costs of various product offerings are assumed to be exogenous. While this assumption is reasonable in situations where a firm produces all of the potential product varieties itself, it does not capture the way a retailers private label capability affects her strategic interaction with the manufacturer of the national brand. As has been recognized in the analytical results of Mills (1995, [30]), Narasimhan and Wilcox (1998, [35]), Groznik and Heese (2007, [17]) and confirmed in the empirical studies of Pauwels and Srinivasan (2004, [38]), among others: the introduction of a private label can help to mitigate double marginalization for the national brand. On the other hand, there may be fixed costs associated with the development of private label capability. Consequently, while a private label product may improve a retailers ability to obtain favorable wholesale prices from a national brand manufacturer, it may not represent an efficient product line extension for the category. This trade-off is somewhat similar to the one studied in Villas-Boas (1998, [51]), but in that analysis the retailer has no ability to produce internally, and instead chooses her product line assortment from among the products offered by the manufacturer.

In the literature, a number of studies have demonstrated how private labels that are perceived to be of lower quality than national brands can mitigate double marginalization in situations in which the private label represents a form of low end vertical differentiation from the national brand. Both Mills (1995, [30]) and Narasimhan and Wilcox (1998, [35]) consider models that demonstrate how the introduction of a low end private label that is produced by the retailer can mitigate double marginalization with respect to a national brand product. Bontems et al. (1999, [6]) also consider the introduction of a low end private label, and allow the quality level of the private label to be controlled by the retailer. They assume that consumer valuations are uniformly distributed and that marginal costs are quadratic in the quality of the product, and focus on two opposing effects that increased private label quality can have upon the wholesale price of the national brand: As private label quality increases, it becomes a better substitute for the national brand, but at the same time, the corresponding increase in marginal costs makes the national brand manufacturer less willing to sacrifice his own margin in order to drive the private label out of the market. Although the manuscript does not consider the product line that would be chosen by a vertically integrated supply chain, the assumption of quadratic costs suggests that it would include a low end product for a wide range of cost parameters.

Several other studies have considered forms of product differentiation other than vertical. For example, Raju et al. (1995, [40]) use an aggregate demand model to demonstrate that a retailer benefits most from the introduction of a private label when the cross price elasticity between the private label and national brands is high and the cross price elasticities among national brands are low. Morton and Zettelmeyer (2004, [33]) allow for horizontal differentiation by modeling two segments of consumers, and they represent the private label as being a low end substitute for the national brands that are offered to each of these segments. They argue that the retailer benefits more from an internally produced private label than from one produced by the national brand manufacturers since it gives the retailer control over its positioning. However, they allow for non-linear contracts that assure that the total channel profit is equal to the first-best profit, and focus on how the private label affects the allocation of channel profit rather than on how it affects the total profit earned by the channel. At least two other papers have modeled the positioning of private label products, e.g. Sayman et al. (2002, [42]) and Choi and Coughlan (2006, [11]), with respect to two partially differentiated national brands.

There has also been a large amount of empirical research devoted to private label products. Those most relevant to our work are the investigations of Ailawadi and Harlam (2004, [1]), and Pauwels and Srinivasan (2004, [38]), both of which focus on how the introduction of a private label affects retail margins and profits within a category. A very nice history of private label products and review of the literature can be found in Steiner (2004, [44]).

The main contribution of our work is to recognize the interaction between two opposing effects of private label development in a decentralized supply chain: On one hand, the retailer's self-interest causes her to introduce private label products that would not be included in the first-best product line for the category. Yet on the other hand, because the development of these inefficient private labels helps to mitigate double marginalization, they can either increase or decrease the total supply chain profit. Our analysis provides insights about how and when the development of a structurally inefficient private label can in fact be beneficial for a supply chain. In an extension to our base model, we allow for the retailer to exert promotional effort to influence the extent to which consumers are exposed to the private label and the national brand. In this analysis, we demonstrate conditions under which the retailer's ability to promote can be either a strategic complement or a strategic substitute for private label capability.

The rest of this chapter is organized as follows. In Section 3.2, we describe the key elements of our base model and also characterize the first best (vertically integrated) solution. In Section 3.3, we model the private label development as a three-stage game in which the retailer first determines whether to develop private label capability, the manufacturer responds with a wholesale price for the national brand, and the retailer responds by setting retail prices. Based on this analysis, we are able to assess how the retailer's opportunity to develop a private label affects the supply chain profit and determine the conditions for which this can be either beneficial or harmful. In Section 3.4 we consider an extension to our basic model in which the retailer can influence the rate at which consumers are exposed to specific products by exerting promotional effort. The main purpose of this extension is to demonstrate that promotional effort and private label development can be either strategic substitutes or strategic complements. Section 3.5 provides a brief summary and discussion of the implications of our analysis.

## 3.2 The Base Model

Consider a supply chain that consists of one manufacturer of a national brand and a single retailer. We adopt the convention of using feminine pronouns for the retailer and masculine pronouns for the manufacturer. In addition, we will henceforth refer to the manufacturer of the national brand as *the manufacturer*.

We consider a product category in which the retailer has the opportunity to develop her own private label product. As is often the case in practice, we assume that the private label product is perceived to be of a lower level of quality than the national brand. This assumption is consistent with those of Mills (1995, [30]), Raju et al. (1995, [40]), and Narasimhan and Wilcox (1998, [35]), among others. Let $q$ denote the exogenous perceived level of quality for the private label product relative to the national brand. Specifically, we assume that the quality of the national brand is equal to one and that

$0 < q < 1$. This is a special case of the demand model presented in Chapter 2 with $\gamma_1 = q$ and $\gamma_2 = 1$. In order to be able to credibly threaten to sell a private label, the retailer must incur a fixed development cost, which we denote by $g$. This may include both the costs of product development as well as the reservation of the capacity. For example, for H-E-B to offer its private label dairy products, it had to incur some fixed costs for determining product specifications, designing packaging, etc., and it also incurs fixed costs for operating its own dairy facilities. In other settings, the national brand manufacturers also produce and supply private label products to retailers. However, such situations give rise to a different set of strategic interactions than those that we seek to address.

Let $C$ and $c$ be the marginal production cost for the national brand and for the retailer's private label respectively. Although other investigations of private label products, e.g. Raju et al. (1995, [40]), normalize the production costs of both the national brand and the private label to zero, we explicitly consider positive production costs to explore the implications of the relative cost difference between the manufacturer and the retailer.

We use the notation $n$ to denote the national brand and $p$ the private label. Thus, product $n$ is corresponding to product 2 and product $p$ corresponding to product 1 in our previous discussion. Similarly, we change the notation $\theta_1$ to $\theta_p$, $\theta_2$ to $\theta_n$, and $\theta_{1,2}$ to $\theta_{np}$. The demand thus can be obtained from Lemma 2.2.2 accordingly. The distribution of consumer valuation $\theta$ is specified in Section 2.3.

We assume that the production costs, $C$ and $c$, of the national brand and the private label as well as the perceived quality level of the private label, $q$, are common knowledge to both the manufacturer and the retailer.

The interactions between the manufacturer and the retailer are represented as a three-stage game. In the first stage, the retailer decides whether to develop private label capability. If she decides to develop the private label, she incurs a fixed cost $g$. Let $d \in \{0, 1\}$ denote the retailer's decision. If $d = 0$, she does not develop the capability, while if $d = 1$, she does. In the second stage, the manufacturer observes the retailer's development decision $d$ and sets a wholesale price $w$ for the national brand. Finally, in stage three, the retailer responds to the wholesale price $w$ by setting retail prices $p_n$ and $p_p$ for the national brand and the private label, respectively. Then, the demand for each product carried by the retailer is realized and profits are collected. Figure 3.1 summarizes the sequence of events in this base setting, and Figure 3.2 depicts the relationship between the two sub-games: $d = 0$, and $d = 1$.

The manufacturer's profit can be represented as:

$$\Pi\left(w, p_n, p_p\right) = Q_n\left(p_n, p_p\right)\left(w - C\right), \tag{3.1}$$

Recall that by taking $p_p \geq q$, we can represent the case where the retailer sells only the national brand. Of course, if the retailer sells only the private label, the manufacturer's profit is zero. For the retailer, we define:

$$\pi\left(w, p_n, p_p\right) = Q_n\left(p_n, p_p\right)\left(p_n - w\right) + Q_p\left(p_n, p_p\right)\left(p_p - c\right). \tag{3.2}$$

The function $\pi(w, p_n, p_p)$ represents the retailer's *net income*, i.e. revenue minus cost of goods sold but excluding any fixed costs. After adjusting the net income to account for fixed costs, the retailer's profit can be written as:

$$\bar{\pi}(d, w, p_n, p_p) = \begin{cases} \pi(w, p_n, q), & \text{if } d = 0; \\ \pi(w, p_n, p_p) - g, & \text{if } d = 1; \end{cases} \tag{3.3}$$

Before we analyze the above three-stage game, let us obtain the first-best solution as a benchmark. This is equivalent to considering the perspective of a *vertically integrated* supply chain that already has the capability of producing a "national brand", and must decide whether to develop a product line extension (private label). In order to examine this, it is useful to introduce the following definition as a means of measuring the *efficiency* of the private label relative to the national brand:

**Definition 3.2.1.** The Ratio of Potential Margin (**RPM**), defined as $\frac{q-c}{1-C}$, represents the ratio between the maximum per-unit margin that could be earned from selling the private label versus that for the national brand.

Recall that the quality of the national brand is equal to one and that the maximum valuation per-unit-of-quality among consumers is $\theta = 1$. Thus, the highest profit margin of the national brand is equal to $1-C$, where $C$ is the per unit production cost of the national brand. Similarly, the highest profit margin of the private label is equal to $q - c$. As a measure of the relative efficiency of the private label, RPM is useful in characterizing the conditions under which a product line extension (private label) will be included in either a vertically integrated or a decentralized supply chain. As shown in the following theorem, the larger the value of RPM, the more attractive the private label becomes to a vertically integrated supply chain as either a product line extension or as a replacement for the national brand.

**Theorem 3.2.1.** *The optimal (first-best) solution to the problem of private label development and pricing for the vertically integrated supply chain can be characterized as follows:*[2]

1. *When* $RPM \leq q$, *then for any development cost* $g$, $d^{FB} = 0$, *the private label is not developed and* $p_n^{FB} = \frac{1+KC}{1+K}$.

2. *When* $q < RPM < 1$, *there exists a threshold value* $\hat{g}_b > 0$, *such that, if and only if* $g < \hat{g}_b$, $d^{FB} = 1$, *the private label is developed, and* $p_n^{FB} = \frac{1+KC}{1+K}$, *and* $p_p^{FB} = \frac{q+Kc}{1+K}$. *Otherwise,* $d^{FB} = 0$, *it is not developed, and* $p_n^{FB} = \frac{1+KC}{1+K}$.

3. *When* $RPM \geq 1$, *there exists a threshold value* $\hat{g}_p > 0$, *such that, if and only if* $g < \hat{g}_p$, *then* $d^{FB} = 1$, *the private label is developed, and* $p_p^{FB} = \frac{q+Kc}{1+K}$ *while the national brand is not sold. Otherwise,* $d^{FB} = 0$, *the private label is not developed and* $p_n^{FB} = \frac{1+KC}{1+K}$.

*Furthermore, the threshold values* $\hat{g}_b$ *and* $\hat{g}_p$ *are both decreasing in* $c$ *and increasing in* $C$.

[2] For those cases in which the private label (national brand) is not sold, the vertically integrated profits are maximized for any $p_p \geq q$ ($p_n \geq 1$). Thus, for these cases, $p_p^{FB} = q$ $\left(p_n^{FB} = 1\right)$ is an optimal solution, even though it is not unique.

When we have $RPM \leq q$, the vertically integrated supply chain does not develop the private label regardless of $g$. When $RPM > q$, then the integrated supply chain develops the private label only if the fixed development cost, $g$, is not too high. Note that, for $q < RPM < 1$, if the private label is developed, it serves as a product line extension and is sold along side the national brand. However, when $RPM \geq 1$, if the private label is developed, it serves as a replacement for the national brand. In those cases for which the first best solution would not include the private label, i.e. when $d^{FB} = 0$, we say that the private label would be a *structurally inefficient* addition to the product line.

## 3.3 Analysis of the Base Model

Our analysis of the three-stage game that is played in the decentralized supply chain follows the standard approach of backward induction. In section 3.3.1, we derive the outcomes that follow the retailer's decision to not develop the private label capability (the sub-game $d = 0$). In section 3.3.2, we derive the corresponding outcomes that follow the retailer's decision to obtain the private label capability (the sub-game $d = 1$). In section 3.3.3, we characterize the retailer's development decision and briefly discuss how this decision might affect the supplier's incentive to reduce his own marginal costs. Finally, in Section 3.3.4, we investigate the impact of the private label on the supply chain profit.

### 3.3.1 Lack of Private Label Capability

In the sub-game ($d = 0$) in which the retailer has no private label capability, the interaction between the national brand manufacturer and the retailer reduces to a classic bilateral monopoly in which the manufacturer acts as a Stackelberg leader. For any wholesale price $w$, the retailer chooses a retail price $p_n$ for the national brand to maximize her profit. Formally, the retailer's problem is:

$$\max_{w \leq p_n \leq 1} \pi\left(w, p_n, q\right) = Q_n\left(p_n, q\right)\left(p_n - w\right). \tag{3.4}$$

Solving problem (3.4) yields the retailer's best response retail price for the national brand as a function of the wholesale price set by the manufacturer:

$$p_n^0\left(w\right) = \frac{1 + Kw}{1 + K}. \tag{3.5}$$

where the superscript 0 indicates that this optimal response is conditional upon $d = 0$, and $K$ is the parameter of the distribution family for consumer valuation.

Anticipating the retailer's best response retail price in (3.5), the manufacturer sets the wholesale price $w$ he will charge the retailer so as to maximize his profit from selling the national brand to the retailer:

$$\max_{C \leq w \leq 1} \Pi\left(w, p_n^0\left(w\right), q\right) = Q_n\left(p_n^0\left(w\right), q\right)\left(w - C\right). \tag{3.6}$$

Performing the maximization in (3.6) yields the wholesale price at which the manufacturer will sell the national brand to the retailer under the case in which the retailer does not develop the private label (where $d = 0$):

$$w^N = \frac{1 + KC}{1 + K}. \tag{3.7}$$

The superscript $N$ ($N$ational brand only) signifies the equilibrium outcome of the sub-game $d = 0$. By substituting the profit-maximizing wholesale price from (3.7) into the expression for the retailer's best response retail price in (3.5) , we can otain the equilibrium retail price and quantity for the national brand:

$$p_n^N = 1 - \frac{K^2 (1 - C)}{(1 + K)^2}, \qquad Q_n^N = \left[ \frac{K^2 (1 - C)}{(1 + K)^2} \right]^K. \tag{3.8}$$

Substituting the wholesale price from (3.7) and the retailer's best response retail price from (3.8) into (3.6) and (3.4) reveals that, in the absence of private label capability, i.e. $d = 0$, the manufacturer's profit and the retailer's net income are:

$$\Pi^N = \frac{1 + K}{K^2} \left[ \frac{K^2 (1 - C)}{(1 + K)^2} \right]^{1+K}, \; \pi^N = \frac{1}{K} \left[ \frac{K^2 (1 - C)}{(1 + K)^2} \right]^{1+K}. \tag{3.9}$$

The equilibrium *net income*[3] for the supply chain of the sub-game $d = 0$, denoted by $\Pi_{SC}^N$, is thus given by:

$$\Pi_{SC}^N = \frac{1 + 2K}{K^2} \left[ \frac{K^2 (1 - C)}{(1 + K)^2} \right]^{1+K}. \tag{3.10}$$

**Theorem 3.3.1.** *When the retailer has no private label capability in a category (the sub-game $d = 0$):*

[3]We use the term "net income" to highlight the fact that this expression does not include fixed costs. When $d = 0$, there are no fixed costs and net income is equal to profit, but the two differ when $d = 1$.

1. *The equilibrium wholesale price $w^N$ and the retail price $p_n^N$ are increasing in the unit cost $C$ of the national brand and decreasing in the distribution parameter $K$.*

2. *The equilibrium sales quantity $Q_n^N$ for the national brand is decreasing in both the unit cost $C$ of the national brand and the distribution parameter $K$.*

3. *The manufacturer's profit $\Pi^N$, the retailer's profit $\pi^N$, and the supply chain profit $\Pi_{SC}^N$ are all decreasing in both the unit cost $C$ of the national brand and the distribution parameter $K$.*

Recall that when the parameter $K$ increases, the mass of the distribution shifts away from high-valuations toward low-valuations. As the mass shifts, both the manufacturer and the retailer are more willing to sacrifice margins on the high valuation consumers to go after a dense set of low valuation consumers. The profits are decreasing because the low-valuation consumers are less valuable than those high valuation consumers due to the same reason.

### 3.3.2 A Private Label in the Category

In this section, we study the equilibrium of the sub-game $d = 1$. Once the retailer has incurred (sunk) the fixed cost to obtain private label capability, she has more options available to her in responding to the manufacturer's wholesale price. Depending on the prices she sets, she can sell only the national brand, both products, or only her private label. Thus, the retailer's problem is:

$$\max_{p_n, p_p} \pi\left(w, p_n, p_p\right). \tag{3.11}$$

Let $p_n^1(w)$ and $p_p^1(w)$ be the retailer's optimal response conditional upon her having private label capability, i.e. $d = 1$.

**Lemma 3.3.2.** *The retailer's best response to wholesale price $w$ for the national brand can be characterized as follows:*

1. *If $0 \le w \le \frac{c}{q}$, the retailer sets $p_n^1(w) = \frac{1+Kw}{1+K}$ and $p_p^1(w) = q$, and she does not sell any private label.*

2. *If $\frac{c}{q} < w < 1 + c - q$, the retailer sets $p_n^1(w) = \frac{1+Kw}{1+K}$ and $p_p^1(w) = \frac{q+Kc}{1+K}$ and she sells both the national brand and the private label.*

3. *If $1 + c - q \le w \le 1$, the retailer sets $p_n^1(w) = 1$ and $p_p^1(w) = \frac{q+Kc}{1+K}$, and she does not sell any national brand.*

*When the private label (national brand) is not sold, the retailer's profits are maximized for any $p_p \ge qp_n^1(w)$ $\left(p_n \ge p_p^1(w) + 1 - q\right)$. Thus, for these cases, $p_p^1 = q$ $(p_n^1 = 1)$ is an optimal solution, even though it is not unique.*

In anticipation of the above retailer pricing response, the manufacturer sets a wholesale price $w$ to maximize his profit:

$$\max_{w} \Pi\left(w, p_n^1(w), p_p^1(w)\right) \tag{3.12}$$

Denote by $w^B$ the equilibrium wholesale price for the sub-game $d = 1$. The superscript $B$ signifies that the retailer has the capability to sell *both* products.

Similarly, define $p_n^B = p_n^1\left(w^B\right)$ and $p_p^B = p_p^1\left(w^B\right)$ to be the equilibrium prices, and define $Q_n^B = Q_n\left(p_n^B, p_p^B\right)$ and $Q_p^B = Q_p\left(p_n^B, p_p^B\right)$ to be the equilibrium quantities for this sub-game.

**Theorem 3.3.3.** *The equilibrium of the sub-game $d = 1$ can be characterized according to the following four mutually exclusive intervals for the RPM (which is defined as $RPM = \frac{q-c}{1-C}$ in Definition 3.2.1): a) $N \equiv \left[0, \frac{Kq}{1+K}\right)$; b) $NPM \equiv \left[\frac{Kq}{1+K}, \frac{Kq}{1+K-q}\right]$; c) $NPH \equiv \left(\frac{Kq}{1+K-q}, 1\right)$; and d) $P \equiv [1, +\infty)$:*

1. *When $RPM \in N$, we have $w^B = \frac{1+KC}{1+K}$, $p_n^B = 1 - \frac{K^2(1-C)}{(1+K)^2}$, $Q_n^B = \left[\frac{K^2(1-C)}{(1+K)^2}\right]^K$ and the private label is not sold.*

2. *When $RPM \in NPM$, we have $w^B = \frac{c}{q}$, $p_n^B = \frac{q+Kc}{q+Kq}$, $Q_n^B = \left(\frac{q+cK}{q+qK}\right)^K$ and the private label is not sold.*

3. *When $RPM \in NPH$, we have $w^B = \frac{1+KC-q+c}{1+K}$, $p_n^B = 1 - \frac{K^2(1-C)+K(q-c)}{(1+K)^2}$, $p_p^B = \frac{q+Kc}{1+K}$, $Q_n^B = \left[\frac{K^2(1+c-C-q)}{(1+K)^2(1-q)}\right]^K$, and $Q_p^B = \left[\frac{K^2(1+c-C-q)}{(1+K)^2(1-q)}\right]^K - \left(1 - \frac{q+cK}{1+K}\right)^K$.*

4. *When $RPM \in P$, the retailer will not sell the national brand in response to any $w > C$. The retailer sets $p_p^B = \frac{q+Kc}{1+K}$ and $Q_p^B = \left(1 - \frac{q+cK}{1+K}\right)^K$, and the national brand is not sold.*

The ranges for the $RPM$ are plotted against the quality level $q$ of the private label in Figure 3.3. The above result generalizes the one obtained by Mills (1995, [30]) by allowing for a more general distribution of consumer valuations and marginal costs for both the private label and national brand.

When the RPM is low, i.e. $RPM \leq \frac{Kq}{1+K}$, the equilibrium wholesale price is independent of $c$, i.e. the private label has no effect upon the strategic interactions between the national brand manufacturer and the retailer. At the other extreme, in region $P$, the private label is so efficient that the retailer cannot be induced to sell the national brand at any wholesale price above the manufacturer's marginal cost. For the two intermediate regions, there are two possibilities: In region $NPM$, the retailer's private label capability causes the manufacturer to reduce his wholesale price by enough to make his national brand sufficiently attractive to the retailer that she sells none of her private label. Notice, that in this region, the wholesale price is independent of the manufacturer's marginal cost $C$. In region $NPH$, the retailer's private label capability still puts pressure on the manufacturer to reduce his wholesale price, but not by enough to discourage the retailer from selling *any* units of her private label. In this region, the equilibrium wholesale price depends upon both $C$ and $c$, reflecting the manufacturer's trade-off between reducing his margin in return for reducing the retailer's substitution of sales of her own private label for sales of the national brand. Consequently, both products are sold in region $NPH$.

From Theorem 3.2.1, the integrated supply chain will not sell the structurally inefficient private label when $RPM \leq q$ for any fixed development cost $g$. Since the lower limit of set NPH is $\frac{Kq}{1+K-q} < q$, the retailer may, for sufficiently small values of $g$, sell the private label even if it is structurally inefficient. Moreover, since the retailer can benefit from lower wholesale prices by developing a private label for even lower levels of $RPM \in NPM$, there may be a wide range of parameters for which the retailer incurs the fixed cost to develop a structurally inefficient private label.

Substituting the equilibrium wholesale price and retail prices into (3.11) and (3.12), we have the equilibrium profits of the manufacturer, $\Pi^B$, and the net income of the retailer and of the supply chain, $\pi^B$ and $\Pi^B_{SC}$ respectively, in the sub-game $d = 1$. The equilibrium is summarized in Table 3.1.

**Corollary 3.3.4.** *The equilibrium of the sub-game $d = 1$ has the following properties:*

1. *Both the wholesale price $w^B$ and the retail price $p_n^B$ of the national brand are increasing in $C$ and $c$, and decreasing in $q$ and $K$.*
2. *The retail price of the private label $p_p^B$ is increasing in $q$ and $c$, and decreasing in $K$.*
3. *The manufacturer's profit $\Pi^B$, the retailer's net income $\pi^B$, and the supply chain's net income $\Pi^B_{SC}$ are all decreasing in $C$.*

From our result in Theorem 3.3.3, we can also see how the distribution of consumer valuations affects the strategic role of the private label. Note that both the threshold, $\frac{Kq}{1+K}$, that separates *low* RPM from *intermediate*, and $\frac{Kq}{1+K-q}$, that separates *intermediate* RPM from *high*, depend on $K$, the parameter of the family $\mathcal{F}^K$ of distribution functions.

**Corollary 3.3.5.** *Both of the thresholds,* $\frac{Kq}{1+K}$*, and* $\frac{Kq}{1+K-q}$*, are increasing in* $K$*.*

As $K$ increases, shifting the concentration of mass toward lower valuations, the retailer's private label must have higher RPM in order to play a strategic role. Although this is a somewhat counter-intuitive result, it is driven by the fact that, when consumers become increasingly concentrated at the low end, the manufacturer will try to pursue them by decreasing his own margin, even in the absence of a private label. Consequently, the private label becomes less attractive to the retailer as a partial substitute for the national brand. In the following Theorem, we make some comparisons between the two pricing sub-games, with and without the private label. Let us define $Q_n^N$ and $Q_n^B$ as the equilibrium sales quantities of the national brand when $d = 0$ and when $d = 1$ respectively.

**Theorem 3.3.6.** *Comparing the equilibrium of the sub-game* $d = 1$ *to that of the sub-game* $d = 0$*, we have the following results:*

1. *The manufacturer charges a lower wholesale price:* $w^B \leq w^N$*.*

2. *There exist a unique threshold value* $c_Q$*, where* $\max\{0, C + q - 1\} \leq c_Q \leq \max\left\{0, \frac{q(1+KC-q)}{1+K-q}\right\}$*, such that* $Q_n^B < Q_n^N$ *when* $0 \leq c < c_Q$ *and* $Q_n^B \geq Q_n^N$ *otherwise.*

3. *The manufacturer achieves a lower profit:* $\Pi^B \leq \Pi^N$*; The retailer and supply chain achieves higher net incomes:* $\pi^B \geq \pi^N$ *and* $\Pi_{SC}^B \geq \Pi_{SC}^N$*;*

*The increase in the retailer's net income is higher than that of the supply chain:* $\pi^B - \pi^N \geq \Pi^B_{SC} - \Pi^N_{SC}$*. In addition, the inequalities are all strict when* $c < \frac{1+KC}{1+K} q$*.*

The result that the private label provides the retailer with additional leverage that can drive down the manufacturer's wholesale price is both intuitive and consistent with the existing results of Mills (1995, [30]), Bontems et al. (1999, [6]), etc., that were obtained for somewhat different assumptions. However, our result has several unique features: First, it captures the way in which marginal production costs and the relative quality of the private label affect the output quantities and the net income / profits. Second, it is explicit in recognizing that the retailer's development of private label capability does not necessarily decrease the sales volume of the national brand. Specifically, when $RPM \in NPM$, the private label capability serves only to provide the retailer with a credible threat to sell the private label. In this region, private label capability encourages the manufacturer to reduce his wholesale price, mitigating double marginalization, yet it does not create any adverse cannibalization effects. Only at higher levels of $RPM$ does the sales quantity of the national brand decrease as a consequence of the retailer's private label capability. Finally, it highlights the fact that the net income of the *supply chain* increases as a result of the retailer's private label capability.

It is of interest to compare the result in part 3 of Theorem 3.3.6 with our earlier result from Theorem 3.2.1 regarding the structural efficiency of the private label. From the earlier result, we know that for any development cost

$g$, the private label would be structurally inefficient when $RPM < q$, which is equivalent to $c > Cq$. This implies that, for $c \in \left(Cq, \frac{1+KC}{1+K}q\right)$, in spite of the fact that the private label would be structurally inefficient for all values of $g$, it would nevertheless increase the total net income of the supply chain. It follows that for small enough development costs, the development of private label capability in such situations could still be a net benefit to the supply chain.

The development of a private label has two opposing effects on the performance of the supply chain: On one hand, the introduction of the private label creates pressure upon the manufacturer to reduce his wholesale price, mitigating *double marginalization*; On the other hand, the private label involves additional development costs and may cannibalize demand for the national brand. In the next section we characterize the conditions under which this trade-off helps to coordinate the decentralized supply chain.

### 3.3.3 Private Label Development Decision

The retailer's optimal decision on private label capability development and the impact of the private label on the supply chain both depend on the value of the fixed development cost $g$. Let us define $\Delta^R = \pi^B - \pi^N$ and $\Delta^{SC} = \Pi^B_{SC} - \Pi^N_{SC}$ . $\Delta^R$ ($\Delta^{SC}$) represents the increase in the retailer's (supply chain's) net income as a result of the retailer's developing private label capability. Obviously, the retailer develops the private label capability if and only if $\Delta^R \geq g$.

**Theorem 3.3.7.** *The optimal solution to the problem of private label development and pricing for the retailer can be characterized as follows:*

1. *When $RPM \in N$, then the private label is not developed ($d^* = 0$) regardless of development cost $g$. The manufacturer sets a wholesale price $w^* = \frac{1+KC}{1+K}$, the retailer sets a retail price $p_n^* = 1 - \frac{K^2(1-C)}{1(1+K)^2}$ .*

2. *When $RPM \in NPM$, there exists a threshold value $\bar{g}_n > 0$, such that, if and only if $g < \bar{g}_n$, then the private label is developed ($d^* = 1$), but it is not sold in equilibrium where $w^* = \frac{c}{q}$, and $p_n^* = \frac{q+Kc}{q+Kq}$. Otherwise, it is not developed ($d^* = 0$), and the prices are $w^* = \frac{1+KC}{1+K}$ and $p_n^* = 1 - \frac{K^2(1-C)}{1(1+K)^2}$.*

3. *When $RPM \in NPH$, there exists a threshold value $\bar{g}_b > 0$, such that, if and only if $g < \bar{g}_b$, then the private label is developed ($d^* = 1$), and the prices are: $w^* = \frac{1+KC-q+c}{1+K}$, $p_n^* = 1 - \frac{K^2(1-C)+K(q-c)}{1(1+K)^2}$, and $p_p^* = \frac{q+Kc}{1+K}$. Otherwise, it is not developed ($d^* = 0$), and the prices are $w^* = \frac{1+KC}{1+K}$ and $p_n^* = 1 - \frac{K^2(1-C)}{1(1+K)^2}$.*

4. *When $RPM \in P$, there exists a threshold value $\bar{g}_p > 0$, such that, if and only if $g < \bar{g}_p$, then the private label is developed ($d^* = 1$) and the retailer sells only the private label at price $p_p^* = \frac{q+Kc}{1+K}$. Otherwise, it is not developed ($d^* = 0$), and the prices are $w^* = \frac{1+KC}{1+K}$ and $p_n^* = 1 - \frac{K^2(1-C)}{1(1+K)^2}$.*

As before, when the private label (national brand) is not sold, it would not be sold for any $p_p \geq qp_n^*$ $\left(p_n \geq p_p^* + 1 - q\right)$. Thus, for these cases, $p_p^* = q$

$(p_n^* = 1)$ *are equilibrium values.*

The above results characterize the retailer's optimal strategy of developing private label capability and pricing of products. Recall from Theorem 3.2.1 that the first best solution will never include the private label when $RPM < q$. However, because $\frac{Kq}{1+K-q} < q < 1$ implies that $q \in NPH$, this is not the case for the decentralized supply chain. At small enough fixed development costs, the decentralized supply chain will include the private label even when $RPM < q$.

From the results presented in Theorem 3.3.7, we can get some insight as to how the development of the private label might affect the incentive for the national brand manufacturer to undertake efforts to reduce his own marginal cost for production and distribution. Recall that, when the retailer lacks private label capability, the manufacturer's profit is $\Pi^N$, as shown in (3.9). In order to ensure that the manufacturer's profit is concave in $C$, we will restrict our attention to $K \leq 1$. Differentiating with respect to $C$, we can see that the marginal effect of production costs on the manufacturer's profit would be:

$$\frac{d\Pi^N}{dC} = -\left[\frac{(1-C)K^2}{(1+K)^2}\right]^K \tag{3.13}$$

On the other hand, if the retailer does have private label capability, then as shown in Theorem 3.3.7, there are several different possibilities: Using the expressions that are shown in Table 1 for the manufacturer's profit when the retailer has private label capability, we can see that:

$$\frac{d\Pi^B}{dC} = \begin{cases} -\left[\frac{K(q-c)}{(K+1)q}\right]^K & \text{for } RPM \in NPM \\ -\left[\frac{K^2(1-C-q+c)}{(K+1)^2(1-q)}\right]^K & \text{for } RPM \in NPH \\ 0 & \text{otherwise} \end{cases} \tag{3.14}$$

1) If the manufacturer's cost, $C$, without the private label is already low enough relative to the quality and cost of the private label that we would have $RPM \in N$, then the development would obviously have no effect upon his incentive to further reduce cost. 2) If the manufacturer's cost, $C$, without the private label would result in $RPM \in NPM$, then the development of the private label would unequivocally increase the manufacturer's incentive to reduce his costs. To see that this is the case, it is easy to see that the absolute value of the upper branch of (3.14) is larger than that for (3.13) so long as $RPM > \frac{qK}{1+K}$, which is a necessary condition to have $RPM \in NPM$. In this range, the private label reduces double marginalization without cannibalizing the manufacturer's demand. Because this results in a larger volume of output for the manufacturer, it increases his incentive to reduce costs. 3) If the manufacturer's cost, $C$, without the private label would result in $RPM \in NPH \cup P$, then the private label may either increase or decrease the manufacturer's incentive to reduce his costs. By comparing the absolute value of the middle branch of (3.14) to that for (3.13), it is easy to see that when $RPM \in NPH$, private label weakens the manufacturer's marginal benefit from cost reduction so long as $\frac{1-q-(C-c)}{1-q} < 1$, which is true so long as $C > c$. When $RPM \in P$, it is obvious that the private label would reduce the

manufacturer's marginal benefit from cost reduction to zero. Thus, when $RPM \in NPH \cup P$, the private label will clearly decrease the manufacturer's marginal incentive to reduce his costs because of the fact that, in this range, the private label is cannibalizing some or all of the demand for the national brand. On the other hand, it may be possible that in this range, the development of the private label would induce the manufacturer to undertake a dramatic reduction in variable costs in order to enable himself to shift the equilibrium into a more favorable range of $RPM$.

### 3.3.4 Impact on Supply Chain Profit

To make a formal comparison between the effect of the private label upon decentralized supply chain profit versus its effect upon the vertically integrated profit, let us define $\Delta^{VI}$ to be the amount by which the vertically integrated supply chain profit would increase if it had the option to produce a product of quality $q$ at marginal cost $c$ given that it already has the capability of producing a product of quality equal to one at marginal cost $C$.

**Theorem 3.3.8.** *For all parameters ($K$, $C$, $c$, and $q$), $\Delta^R \geq \Delta^{VI}$. Thus, if the private label is structurally efficient, i.e. $\Delta^{VI} \geq g$, then the retailer will always develop private label capability, but she may also develop the capability to produce some structurally inefficient private labels.*

Obviously, the retailer develops the private label only when it is in her best interest to do so, i.e. $\Delta^R \geq g$. For the supply chain to benefit from the development of private label capability, we additionally need $\Delta^{SC} \geq g$.

From part c of Theorem 3.3.6, we know that $\Delta^R \geq \Delta^{SC}$, which implies that the development of these private labels may or may not be beneficial to the supply chain. However, it is important to note the distinction between $\Delta^{SC}$ and $\Delta^{VI}$: While both are measures of the impact of the private label upon the net income of the total supply chain, $\Delta^{SC}$ assumes that prices are set in a decentralized fashion, i.e. the manufacturer sets a wholesale price for the national brand and the retailer responds with retail prices for both the national brand and the private label. In contrast, $\Delta^{VI}$ assumes that the supply chain is vertically integrated so that there is no need for a wholesale price. Because of this distinction, it is quite possible that a structurally inefficient private label, i.e. one for which $\Delta^{VI} < g$, could nevertheless have a beneficial effect upon the performance of the decentralized supply chain, i.e. $\Delta^{SC} \geq g$.

In Figure 3.4, we illustrate the conditions under which the decentralized supply chain will include the private label as a product line extension when the first-best solution would not. In the figure, we show the threshold development cost ($\bar{g}$ ) below which the private label would be developed in the decentralized supply chain under several different sets of parameters. We compare this threshold to both $\Delta^{SC}$, the increase in the net income of the decentralized supply chain, and to $\hat{g}$ the threshold development cost below which the first-best solution would include the private label.[4] In Figures 3.4a - 3.4c, we take $q = 0.6$, $C = 0.5$, and allow $c$ to vary between 0 and $q = 0.6$. To investigate the

[4]Recall from Theorem 3.2.1 that the threshold development costs below which the private label is included in the first-best solution is either $\hat{g}_b$ or $\hat{g}_p$, depending upon the value of $RPM$. In the figures, we adopt the un-subscripted, $\hat{g}$, to represent both of these.

sensitivity to the distribution of consumer valuations, we consider: $K = 0.5$ (concentration of mass at high valuations), $K = 1$ (uniform distribution), and $K = 2$ (concentration of mass at the low valuations). In comparing the plots for different values of $K$, it is important to recognize that the scale of the vertical-axes varies in order to best highlight the different regions for each set of parameters. Figures 3.4d - 3.4f are similar, but here we hold the retailer's cost constant at $c = 0.4$, and allow the manufacturer's cost $C$ to vary between 0 and 1.

In all six of the figures, we have labeled four different regions. Regions $I$ and $IV$ represent two extremes. Region $I$ is the only region in which the decentralized retailer would not develop private label capability, and region $IV$ is the only region in which the the private label is structurally efficient, and would be included as a product line extension in the first best solution. In both of the intermediate regions, $II$ and $III$, the private label is structurally inefficient, but would be developed in the decentralized supply chain. In region $III$, the development of the structurally inefficient private label would nevertheless result in an increase in the total decentralized supply chain profits. In this region, the private label's ability to mitigate double marginalization more than compensates for its fixed costs and cannibalization of the demand for the national brand. By developing the private label, the retailer gains more than the manufacturer loses. On the other hand, in region $II$, the retailer also benefits from developing the capability to produce a structurally inefficient private label, but in this region, she gains less than the manufacturer loses, and the development of the private label adversely affects the total decentralized supply chain profits.

It is also of some interest to observe how the shape of these regions changes in response to the parameters. First, we can see that as $K$ increases, i.e. shifting the concentration of consumer valuations toward the low end, region $I$ expands, primarily due to the shrinkage of regions $II$ and $III$, so that the private label is developed for a smaller set of parameters. This suggests that private labels may be least effective in product categories in which only a very small number of consumers have high valuations, whereas they may be most effective in categories for which relatively few consumers have very low valuations.

It is also of interest to observe the sensitivity with respect to the marginal production costs. Recall from Theorem 3.2.1 that the threshold development cost ($\hat{g}_b$ or $\hat{g}_p$), below which the first-best solution includes the private label, is decreasing in the cost $c$ of the private label and is increasing in the cost $C$ of the national brand. This is quite intuitive since from the perspective of a vertically integrated supply chain, the product line extension is most attractive when its marginal cost is low relative to that of the original (national brand) product. However, in the decentralized supply chain, the thresholds are not necessarily monotone in $c$ and $C$. In Figures 3.4a - 3.4c, we can see that although the retailer's development cost threshold, $\bar{g}$, is monotone decreasing in $c$, the threshold, $\Delta^{SC}$, below which the supply chain benefits is not. It first decreases, then increases, before decreasing again. (The increase is subtle, but we can show this analytically.) The reason for the slight increase can be explained in terms of the trade-off between double marginalization and cannibalization. At the point where $\Delta^{SC}$ attains a local minimum, the ratio of cost to quality for the private label is equal to that of the national brand, so that the retailer is very willing to substitute her own private label for the national brand. In this region, a small increase in the retailer's cost causes her to substitute less of the increasingly inefficient private label.

The lack of monotonicity with respect to the manufacturer's cost, $C$, is much more dramatic. In figures 3.4d - 3.4f, we can see that once the thresholds $\bar{g}$ and $\Delta^{SC}$ begin to take positive values, they both are first increasing, then decreasing, and then increasing in $C$. The reason for this can be explained as follows: When the thresholds initially become positive, the cost of the national brand, $C$, is still low enough that the only role played by the private label is to provide the retailer with leverage to cause the manufacturer to reduce his wholesale price. Since the retailer does not actually sell her private label at these values of $C$, its only role is to mitigate double marginalization. As $C$ increases, the private label becomes an increasingly attractive substitute for the national brand. At first, in the region for which $\bar{g}$ and $\Delta^{SC}$ are still increasing, this serves only to put stronger pressure on the manufacturer's wholesale price. However, at the point at which $\bar{g}$ and $\Delta^{SC}$ attain a local maximum (at roughly $C = 0.4$, $C = 0.52$, and $C = 0.6$ in figures 3.4d, 3.4e, and 3.4f respectively), the manufacturer is no longer willing to set his wholesale price low enough to completely discourage sales of the private label. Beyond this point, the retailer begins to substitute small amounts of her private label for the national brand, even though it is initially structurally inefficient. As a consequence of the retailer's substitution of a structurally inefficient private label for the national brand, the net income of both the retailer and the supply chain then decrease in $C$ until they attain local minima at approximately the point at which the private label becomes structurally efficient, i.e. where $\hat{g}$ becomes positive. Beyond this point of local minima, the private label becomes an increasingly attractive substitute for the national brand, eventually becoming the only product that is sold, even by the vertically integrated supply chain.

## 3.4 Retailer's Promotional Effects

Thus far, our analysis has focused on the strategic role that private label development can play in the interactions between a retailer and a national brand manufacturer, and we have characterized the conditions under which this role can be either beneficial or detrimental to the total profits of the decentralized supply chain. To highlight the role of the private label, we have considered it as the retailer's only strategic lever with respect to the manufacturer. However, in practice, a retailer's ability to direct her promotional effort may be another important strategic lever.

Within the literature on private label products, relatively little attention has been paid to the effect of either retailer or manufacturer initiated demand stimulation. Soberman and Parker (2006, [43]) consider the effect of national brand advertising, but to our knowledge no one has considered a retailer's ability to influence retail demand and its implications for private label introduction.

Outside of the private label literature, a number of papers, e.g. Tsay and Agrawal (2000, [48]); Taylor (2002, [46]); Krishnan et al. (2004, [24]); Iyer et al. (2005, [21]); Xia and Gilbert (2007, [52]), etc., have allowed for retailer effort that stimulates demand for a product that is obtained from a manufacturer. However, none of these have allowed for the retailer having the ability to offer her own low end version of the product. In this section, we extend our original model in order to analyze the interactions between a retailer's development of a private label and her ability to influence the demand for a product via promotional effort.

In modeling these interactions, we want to capture several important operational dynamics of promotional effort at the retail level. First, we want to capture the fact that much of a retailer's in-store promotional effort is aimed at influencing which products a consumer is exposed to once he or she enters the store. For example, by positioning a product at the end of an aisle or by allocating additional shelf space to a product, a retailer can ensure that a larger portion of all consumers in the store will be exposed to it. These activities can be particularly effective for products that are *impulse purchases*, for which the consumer makes the decision to purchase after entering the store. A second dynamic that we want to capture is the fact that although a retailer may direct her promotional effort disproportionately among the products in a category, there may be spillover effects. For example, once a retailer gets a consumer to notice one product in a category, it may be relatively easier for her to get that consumer to notice other products in the category. Finally, we want to capture the fact that promotional effort tends to have diminishing returns so that continuing to increase promotional effort results in smaller and smaller incremental increases in consumer exposure.

To capture these dynamics, we assume that there is some base number (normalized to one) of consumers who would be exposed to the product category even in the absence of any promotional effort on the part of the retailer. However, by exerting promotional effort, the retailer can expand the number of consumers who are exposed to the product by some percentage. Let $x$ and $y$ denote the percentage increase in consumer exposure received by the national brand and the private label as a result of the retailer's promotional effort, where $x$ and $y$ can be different. When $x = y$, the retailer promotes the whole category and the number of consumers who will be exposed to both products increases to $1 + x$. When $x < y$, the retailer promotes the private label more than the national brand. The number of consumers who will be exposed to both products increases to $1 + x$, and $y - x$ additional consumers will be exposed to the private label only. Similarly, when $x > y$, the number of consumers who will be exposed to both products becomes $1 + y$ and an additional $x - y$ will be exposed to the national brand only. The cost of increasing the consumer exposure levels by $x$ and $y$ is represented as follows:

$$C(x, y) = \frac{1}{2} ra (\min\{x, y\})^2 + \frac{1}{2} a (\max\{x, y\})^2, \tag{3.15}$$

where, $a > 0$ and $0 < r < 1$. The parameter $a$ reflects the difficulty of increasing consumer exposure to products in a category. Larger values of $a$ indicate that it is more difficult to increase sales in a category through increased consumer exposure. The parameter $r$ reflects the extent of promotional synergies due to spillover effects. When $r = 0$, we have *free-riding* in the sense that, once a group of consumers has been exposed to one product in a category, there is no additional cost for exposing them to a second product. When $r = 1$, we have the opposite extreme of no promotional synergies due to spillover effects. For this case, the promotion of the two products are completely independent. For intermediate values of $r$ there are varying degrees of spill-over synergies. For these cases, once a consumer is exposed to one product in the category, it is cheaper, but not costless, to get him or her to notice another product.

To facilitate analysis, we assume that the distribution of valuations among the additional consumers ($x$ and $y$) is identical to that for the original base set of consumers. While we admit that this is somewhat restrictive, it may not be unreasonable for products that are impulse purchases or for which consumers purchase them regularly but not every time they visit the store.

When we include promotional effects as described above, the retailer's profit is separable in pricing and promotion. This is a consequence of our implicit assumption that the additional consumers who are exposed to a product as a result of promotional activity have the same distribution of valuations as do the base set of consumers who would have been exposed to it without any promotion.

As a result of this separability, the first best solution, i.e. what a vertically integrated supply chain would do, has some similarity to the first best solution for our original model. Specifically, the vertically integrated supply chain would not develop a private label when $RPM < q$, regardless of how low the development cost was. However, for $q < RPM < 1$ the promotional effort would affect the increase in net income as a result of having the private label and this changes the threshold development costs. Within the interval $q < RPM < 1$, the vertically integrated supply chain's promotional strategy can be characterized according to three sub-intervals. For $RPM$ in the lower sub-interval it promotes the national brand more than the private label (i.e. product line extension); for $RPM$ in the intermediate sub-interval, it promotes both products equally; and for $RPM$ in the upper sub-interval, it promotes the private label more than the national brand. Because the analysis of this vertically integrated solution does not yield much additional insight, we omit the details. Similarly, the analysis of the sub-game in which the retailer does not develop the private label ($d = 0$) is a relatively straight-forward extension of the analysis in Section 3.3.1, and this too is omitted.

To demonstrate the interaction between the two strategic levers that are available to the retailer, i.e. promotion and private label development, we focus our attention on the sub-game ($d = 1$) in which the retailer develops the private label. As a consequence of the fact that the retailer's profit is separable in pricing and promotion, the retailer's optimal pricing response to a given wholesale price continues to involve $p_n(w) = \frac{1+Kw}{1+K}$ and $p_p(w) = \frac{q+Kc}{1+K}$

as defined in Lemma 3.3.2 and the regions $R_N$, $R_B$, and $R_P$ as defined in (2.6) ($R_N = R_2$, $R_P = R_1$, and $R_B = R_{1,2}$). Thus, depending on the value of $d$, the retailer's profit $\pi\left(d, w, p_n, p_p, x, y\right)$ can be written as:

$$\begin{aligned}
\bar{\pi}\left(0\right) &= \pi(w, p_n, q) - C\left(x, 0\right) \\
\bar{\pi}\left(1\right) &= \begin{cases} \pi(w, p_n, p_p)\left(1 + x\right) + \pi(w, 1, p_p)(y - x) - C\left(x, y\right) - g & \text{if } y \geq x \\ \pi(w, p_n, p_p)\left(1 + y\right) + \pi(w, p_n, q)(x - y) - C\left(x, y\right) - g & \text{if } x \geq y \end{cases}
\end{aligned}$$

where $\bar{\pi}\left(0\right) = \bar{\pi}\left(0, w, p_n, p_p, x, y\right)$ and $\bar{\pi}\left(1\right) = \bar{\pi}\left(1, w, p_n, p_p, x, y\right)$. Borrowing and extending the notation from Section 3.3, let $x^0(w)$ and $p_n^0(w)$ as the retailer's optimal promotional and pricing responses conditional upon $d = 0$. It is easy to show that:

$$p_n^0(w) = \frac{1 + Kw}{1 + K} \qquad\qquad x^0(w) = \frac{1}{a}\pi\left(w, p_n^0, q\right)$$

Similarly, let $x^1(w)$ , $y^1(w)$, $p_n^1(w)$, and $p_p^1(w)$ as the retailer's optimal promotional and pricing responses conditional upon $d = 1$.

**Theorem 3.4.1.** *For the case when the retailer has private label capability, $d = 1$, there exist four constants $w_1$, $w_2$, $w_3$, and $w_4$, such that the retailer's optimal pricing and promotional strategies can be characterized as follows, where $\pi\left(w, p_n, p_p\right)$ is defined in equation (3.2):*

1. *When $0 \leq w \leq w_1$, the retailer promotes and sells only the national brand by setting $p_n^1\left(w\right) = \frac{1+Kw}{1+K}$ and $x^1\left(w\right) = \frac{1}{a}\pi\left(w, p_n^1, q\right)$. For the private label, she sets $y^1\left(w\right) = 0$ and she is indifferent among all $p_p \geq q$.*

2. *When $w_1 < w < w_2$, the retailer promotes and sells both products. She sets the prices to $p_n^1(w) = \frac{1+Kw}{1+K}$ and $p_p^1(w) = \frac{q+Kc}{1+K}$, and she promotes the national brand more than the private label by setting $x^1(w) = \frac{1}{a}\pi\left(w, p_n^1, p_p^1\right) > y^1(w) = \frac{1}{ra}\left[\pi\left(w, p_n^1, p_p^1\right) - \pi\left(w, p_n^1, q\right)\right]$.*

3. *When $w_2 \leq w \leq w_3$, the retailer promotes and sells both products. She sets the prices to $p_n^1(w) = \frac{1+Kw}{1+K}$ and $p_p^1(w) = \frac{q+Kc}{1+K}$, and she promotes the two products equally by setting $x^1(w) = y^1(w) = \frac{1}{(1+r)a}\pi\left(w, p_n^1, p_p^1\right)$.*

4. *When $w_3 < w < w_4$, the retailer promotes and sells both products. She sets the prices to $p_n^1(w) = \frac{1+Kw}{1+K}$ and $p_p^1(w) = \frac{q+Kc}{1+K}$, and she promotes the national brand less than the private label by setting $x^1(w) = \frac{1}{ra}\left[\pi\left(w, p_n^1, p_p^1\right) - \pi\left(w, 1, p_p^1\right)\right] < y^1(w) = \frac{1}{a}\pi\left(w, 1, p_p^1\right)$.*

5. *When $w_4 \leq w \leq 1$, the retailer promotes and sells only the private label by setting $p_p^1(w) = \frac{q+Kc}{1+K}$ and $y^1(w) = \frac{1}{a}\pi\left(w, 1, p_p^1\right)$. For the national brand, she sets $x^1(w) = 0$ and she is indifferent among all $p_n \geq 1$.*

*As before, when the private label is not sold, it would not be sold for any $p_p \geq qp_n^1(w)$, so $p_p^1 = q$ is an optimal response. Similarly,when the national brand is not sold, it would not be sold for any $p_n \geq p_p^1(w) + 1 - q$, so that $p_n^1 = 1$ is an optimal response.*

The above result demonstrates how the retailer can use promotional effort to either substitute for or to complement her private label capability. Note that from the perspective of the retailer, the RPM is given by $\frac{q-c}{1-w} \geq \frac{q-c}{1-C}$

since the maximum margin on the national brand is $1-w$ for the retailer. Thus, as $w$ increases, the relative efficiency of the private label increases in the eyes of the retailer. From Theorem 3.4.1, we can see that as $w$ increases enough that the retailer begins selling the private label, she initially promotes it less than the national brand. In this range, $w < w_2$, the private label is less efficient than the national brand, and its main role is to encourage the manufacturer to lower his wholesale price. Since the retailer's ability to promote the national brand also encourages the manufacturer to offer a lower wholesale price, the private label is a strategic substitute for promotion. As the wholesale price increases further to $w > w_3$, the retailer promotes the private label more than the national brand, eventually promoting and selling only the private label when $w > w_4$. In this range, the private label becomes more efficient than the national brand, and the retailer's ability to promote it amplifies the increase in her net income that she receives as a result of having private label capability. Thus, promotion serves as a strategic complement to private label capability.

As we did in our original model, we can again express the manufacturer's profit as a function of his wholesale price, $w$, taking into account the retailer's best pricing and promotional response. When $d = 0$, the manufacturer's profit will be $\Pi(w) = (1 + x^0(w))\, \Pi\left(w, p_n^0(w), q\right)$. When $d = 1$, the manufacturer's profit will be:

$$\Pi(w)=\begin{cases}[1+x^1(w)]\,\Pi\left(w,p_n^1(w),q\right), & if\ 0\le w\le w_1;\\ [x^1(w)-y^1(w)]\,\Pi\left(w,p_n^1(w),q\right)+ & \\ [1+y^1(w)]\,\Pi\left(w,p_n^1(w),p_p^1(w)\right), & if\ w_1\le w\le w_2;\\ [1+x^1(w)]\,\Pi\left(w,p_n^1(w),p_p^1(w)\right), & if\ w_2\le w\le w_3;\\ [1+x^1(w)]\,\Pi\left(w,p_n^1(w),p_p^1(w)\right), & if\ w_3\le w\le w_4;\\ 0, & if\ w_4\le w\le 1.\end{cases}\tag{3.16}$$

Although we have not been able to obtain a closed form solution for the equilibrium value of $w$ for this sub-game, it can be found numerically. We can then determine the threshold development costs just as we did for our original model. Recall that $\hat{g}$ and $\bar{g}$ denote the threshold development costs below which the private label would be developed by the vertically integrated supply chain and by the decentralized supply chain respectively. Recall also, that $\Delta^{SC}$ denotes the increase in the net income of the decentralized supply chain that results from the retailer having private label capability.

To illustrate the strategic interaction between private label capability and promotion, we replicated the numerical analysis from Figure 3.4, this time including promotional effects, and the results are presented in Figure 3.5. As before, in plots a-c, we take $C = 0.5$, and allow $c$ to vary for $K = 0.5$, $K = 1$, and $K = 2$. Similarly, in plots d-f, we take $q = 0.6$, $c = 0.4$ and allow $C$ to vary for $K = 0.5$, $K = 1$, and $K = 2$. We take the promotional parameters to be $a = r = 0.1$ throughout. These parameter values imply that promotion is relatively effective. For example, for the case in which $q = 0.6$, $c = 0.4$ , $C = 0.5$, and $K = 1$, the equilibrium effect of promotion was to increase consumer exposure to the products by about 50%. For smaller values of $a$, promotional effects obvioulsy create a larger distortion of our original results, while for larger values, the effects of promotion gradually disapear. On the other hand, varying the value of the promotional synergy parameter, $r$, has relatively little effect on the fundamental structure of the results shown in Figure 3.5.

Let us now compare Figure 3.5 to Figure 3.4. First, we note that the intercept of $\hat{g}$ with the horizontal axis, which represents $c$ or $C$, is not affected by promotional effects. This is a reflection of the separability of pricing and promotion so that promotion does not affect the minimum value of $RPM$ for which the vertically integrated supply chain would develop the product line extension even with zero development costs. To assess the strategic interactions between private label capability and promotion, let us begin by comparing the regions for which $\bar{g} > 0$ in Figures 3.5a - 3.5c to those in Figures 3.4a - 3.4c. In all cases, when the retailer can promote, the range of $c$ for which $\bar{g} > 0$ contracts. Specifically, private label capability stops increasing the retailer's net income at lower values of $c$ when she can promote. This is most evident when $K = 0.5$, where consumer valuations are concentrated at the high end, but it occurs in all three cases. This implies that for the largest values of $c$ for which the private label would be developed, i.e. when $RPM \in NPM$, the ability to promote decreases the retailer's proclivity to develop the private label. For further evidence of this, we can compare the minimum values of $C$ for which $\bar{g} > 0$ in Figures 3.5d - 3.5f versus in Figures 3.4d - 3.4f. Again, the ability to promote causes the range of $C$ for which $\bar{g} > 0$ to contract. At the smallest values of $C$ for which the private label would be developed, i.e. when $RPM \in NPM$, the ability to promote decreases the retailer's proclivity to develop the private label. These comparisons suggest that for private labels that are relatively inefficient as substitutes for the national brand, the ability to develop a private label is a strategic substitute for the ability to influence demand through promotion.

On the other hand, let us consider how the two strategic levers interact when the private label is efficient, which is the case in the figures when either $C$ is large or $c$ is small. Comparing the value of $\bar{g}$ for the lowest values of $c$ in Figures 3.5a - 3.5c to those in Figures 3.4a - 3.4c, it is quite clear that, for these cases of very efficient private labels, as measured by large $RPM$, promotional ability increases the retailer's proclivity to introduce the private label. Similarly, by comparing the value of $\bar{g}$ for the largest values of $C$ in Figures 3.5d - 3.5f to those in Figures 3.4d - 3.4f, which are again indicative of high $RPM$, promotional ability increases the retailer's proclivity to introduce the private label.

These comparisons indicate that the nature of the strategic interaction between private label development and promotion depends upon the relative efficiency of the private label. When $RPM$ is small and the private label is relatively inefficient, the two are strategic substitutes. On the other hand, when $RPM$ is high, and the private label is efficient, then the two are strategic complements; the ability to promote only magnifies the benefits from developing the private label.

Finally, the discontinuities in both $\bar{g}$ and $\Delta^{SC}$ that appear in Figure 3.5 deserve some discussion These discontinuities are the consequence of how the retailer allocates her promotional effort as $w$ increases. Recall from Theorem 3.4.1 that as $w$ increases, the retailer shifts from allocating more promotional effort to the national brand to allocating more effort to the private label. The discontinuities are the result of how the manufacturer anticipates this response in equilibrium.

## 3.5 Conclusions

The main contribution of our work is to recognize the role that is played by a retailer's ability to develop and produce her own private label product in coordinating a decentralized supply chain. In contrast to the existing literature, we explicitly consider the fact that development costs are often a prerequisite for a retailer selling her own private label, and we also recognize that a retailer's marginal costs may differ from those of a national brand manufacturer, both absolutely and relative to the qualities of their products.

By comparing a private label in a decentralized supply chain to a product-line extension in a vertically integrated supply chain, we obtain a working definition of a *structurally efficient private label* as one that would be developed as a product line extension by a vertically integrated supply chain. We then establish that, in a decentralized supply chain, the retailer will develop structurally efficient private labels (if she has the opportunity to do so), but she will also develop some structurally inefficient private labels. While the development of these structurally inefficient private labels always reduces the profit of the national brand manufacturer, they may or may not lead to higher overall supply chain profits. When development costs are low enough, the retailer benefits more from developing the private label than the manufacturer loses, so that their combined profits increase. Thus, even though the private label would not have been developed by a vertically integrated supply chain as a product line extension, it nevertheless is developed by a decentralized supply chain and furthermore actually serves to increase the overall profit. For these cases, the structural inefficiency of the private label is dominated by its mitigating effects upon double marginalization. However, at higher (but not too high) development costs, the retailer develops structurally inefficient private labels that benefit her less than they harm the manufacturer. In these cases, the fixed cost and the adverse cannibalization effects of the private label more than offset its mitigating effects upon double marginalization.

In an extension to our base model, we endow the retailer with the additional strategic lever of being able to exert promotional effort to increase the number of consumers who are exposed to the private label and/or the national brand. This analysis yields the insight that the strategic interaction between private label development and promotional effort depends upon the relative efficiency of the private label. When the private label is efficient, i.e. its cost to quality ratio is low, the ability to exert promotional effort serves only to enhance its attractiveness. On the other hand, when the private label is less efficient, i.e. it has a higher cost to quality ratio, then private label development and the promotional effort are strategic substitutes. For these cases, the private label is developed primarily to create pressure on the manufacturer to lower his wholesale price. But because the retailer's ability to exert promotional effort also puts downward pressure on the wholesale price, promotional effort serves as a strategic substitute for private label development in these cases.

Finally, we have obtained all of our results for a family of distributions of consumer valuations that is more general than the uniform. While this is itself a technical contribution, the main benefit is that it has allowed us to glean some useful insights into how the distribution of consumer valuations affects the role that is played by a private label. Somewhat contrary to intuition, the role of the private label is most significant, in terms of its impact on both the retail and supply chain profits, when consumer valuations are concentrated at the high end of the spectrum.

| | $\pi^B$ | $\Pi^B$ | $\Pi^B_{SC}$ |
|---|---|---|---|
| $RPM \in N$ | $\frac{1}{K}\Delta_1 - g$ | $\frac{1+K}{K^2}\Delta_1$ | $\frac{1+2K}{K^2}\Delta_1 - g$ |
| $RPM \in NPM$ | $\frac{1}{K}\Delta_2 - g$ | $\frac{(1+K)(c-qC)}{K(q-c)}\Delta_2$ | $\frac{q(1-C)+K(c-qC)}{K(q-c)}\Delta_2 - g$ |
| $RPM \in NPH$ | $\frac{q}{K}\Delta_2 + \frac{1-q}{K}\Delta_3 - g$ | $\frac{(1+K)(1-q)}{K^2}\Delta_3$ | $\frac{q}{K}\Delta_2 + \frac{(1+2K)(1-q)}{K^2}\Delta_3 - g$ |
| $RPM \in P$ | $\frac{q}{K}\Delta_2 - g$ | 0 | $\frac{q}{K}\Delta_2 - g$ |

Table 3.1: The equilibrium profit of the retailer, the manufacturer, and the supply chain when the retailer has developed the private label capability (where $d = 1$), where, $\Delta_1 = \left[\frac{K^2(1-C)}{(1+K)^2}\right]^{1+K}$, $\Delta_2 = \left[\frac{K(q-c)}{(1+K)q}\right]^{1+K}$, and $\Delta_3 = \left[\frac{K^2(1-C+c-q)}{(1+K)^2(1-q)}\right]^{1+K}$.

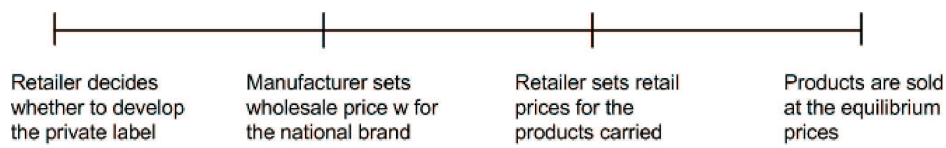



Figure 3.1: Timing in the base model.

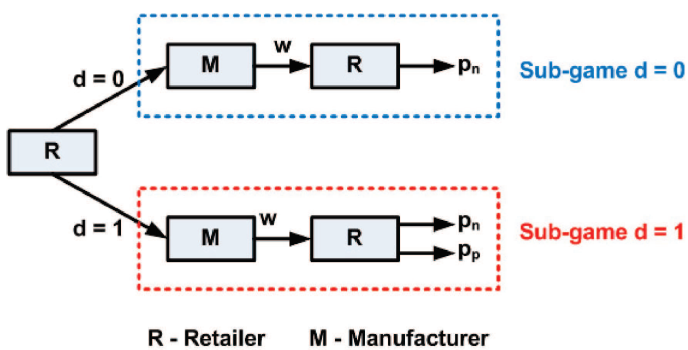



Figure 3.2: The structure of the Stackelberg game and the two sub-games $d = 0$ and $d = 1$.

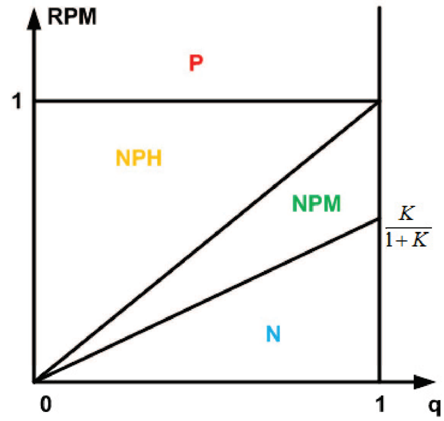
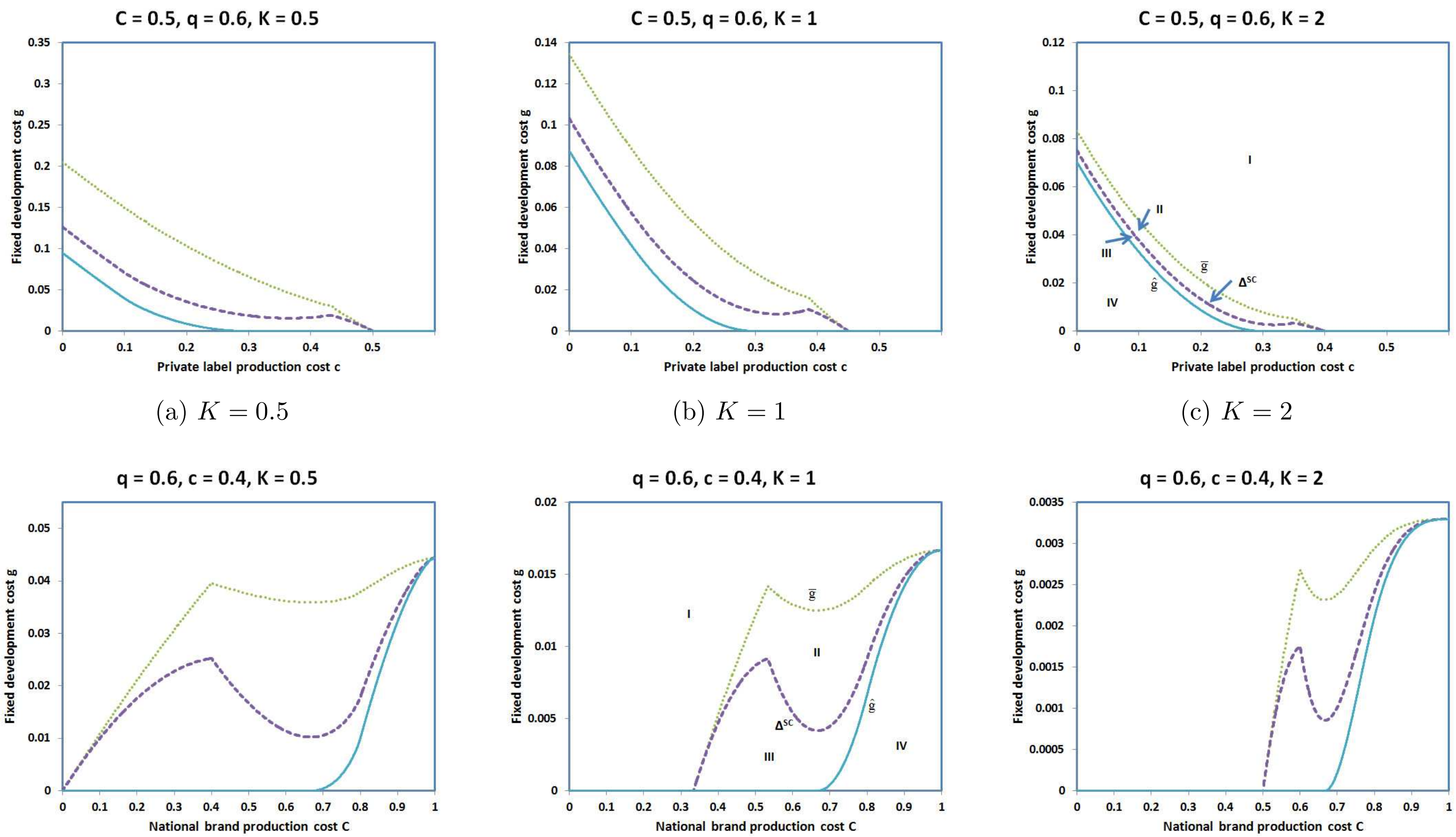


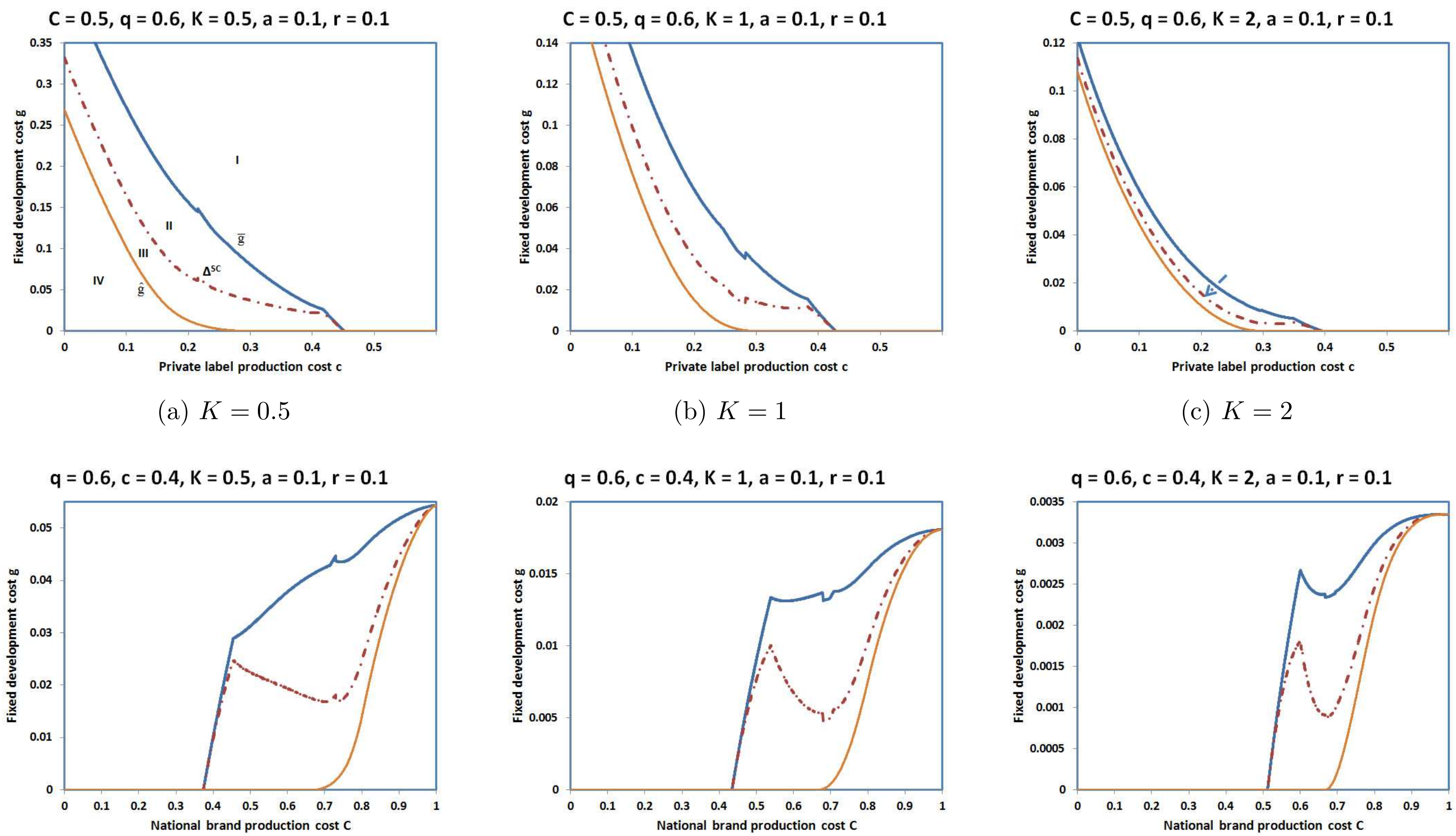

Figure 3.3: The ranges of $RPM$ values plotted against the quality level $q$ of the private label for each type of equilibrium.




(a) $K = 0.5$ (b) $K = 1$ (c) $K = 2$

(d) $K = 0.5$ (e) $K = 1$ (f) $K = 2$

Figure 3.4: The retailer's private label capability development decision plotted in $(c, g)$ and $(C, g)$ spaces without promotional effects.




(a) $K = 0.5$ (b) $K = 1$ (c) $K = 2$

(d) $K = 0.5$ (e) $K = 1$ (f) $K = 2$

Figure 3.5: The retailer's private label capability development decision plotted in $(c, g)$ and $(C, g)$ spaces with promotional effects.

# Chapter 4

# Competition and Product Line Efficiency

## 4.1 Introduction

Since Apple Inc. introduced its first portable media player, iPod, in October 2001, Apple has developed several generations, from the iPod Classic version to the most recent iPod Shuffle. As of September 2008, more than 173 million iPods have been sold worldwide, making iPod the best-selling digital audio player series in history. However, the high product margin has attracted numerous competitors to this new market, among them the Walkman MS series which was first introduced by Sony Corporation at the end of 2003, and the Microsoft Zune which appeared first in November 2006. Regarding manufacturing, numerous components such as CPUs, hard drives, batteries, etc. are needed to make these portable media players. Among all those components, some critical components are usually supplied by a small number of suppliers. These components are normally made by suppliers such as Toshiba for hard drives, Sony for batteries, and Freescale Semiconductor for chips, rather than by the producers of the audio players. In PC industry, firms also share common critical components from the same supplier. For example, All windows based PCs are using Microsoft Windows operating systems, whether they are high end work station computers or low end personal computers.

With an incumbent original equipment manufacturer (OEM) such as Apple and a strategic supplier such as Toshiba or Freescale to supply different components, it is interesting to see how the entry of a rival such as Microsoft affects the performance of the incumbent, and whether the incumbent OEM's capacity investment has any impact on the interactions with the supplier and the rival OEM. Without strategic suppliers, obviously the incumbent will suffer as its demand is cannibalized by the rival OEM. However, the effect on the incumbent OEM's performance is not clear when a strategic supplier is present, because with the strategic supplier who sells a critical component to both the incumbent OEM and the rival OEM, the role of competition may change. On one hand, the rival OEM's product cannibalizes the demand of the incumbent. On the other hand, in order to induce the rival OEM to enter the market, the component supplier may have incentives to accept lower margins, particularly if the rival either has higher production costs or lower quality than the incumbent. In this sense, competition may actually enhance a firm's strategic positioning with respect to the supplier.

We also notice that firms intentionally create inefficiency in their product line by limiting the availability of their higher margin, popular products. For example, many brands today introducing limited edition products as part of their product line. Limited edition products have been used in a lot of industries such as instruments (e.g., Steinway pianos), automobiles and in fashion goods. A motivation for bringing inefficiency to product line that is often discussed in literature are to create scarcity by limiting the quantity of a product. For the competitive implications, Balachander and Stock (2009, [4]) study the question of when and when not to offer the limited edition products. However, there has been little discussion of the strategic interaction between competition and product line inefficiency when competing firms facing a common supplier. In this paper, we investigate the strategic effect of competition and product line inefficiency in a supply chain setting where the competing firms facing a supplier of common components. Especially, we answer the following research questions: If an OEM faces potential entry of a rival, how will this affect its profit? How does the OEM's capacity commitment affect the wholesale price offer by the supplier? How does the OEM balance the strategic effects of competition and capacity commitment when facing a common supplier?

Motivated by the above interaction between the strategic supplier and a potential rival OEM, even in the absence of competition, the incumbent OEM may be able to improve her strategic positioning with respect to the supplier by vertically differentiating her product line and limiting the amount of capacity that can be used to produce the high-end product. More specifically, when the quality of the low-end product is not too low and the incumbent can credibly signal the supplier that she will only produce a limited number of high-end products, the strategic supplier can either charge a high wholesale price to the incumbent–in which case the incumbent buys a limited quantity to only produce the high-end product, or charge a low wholesale price to induce the incumbent to buy a larger quantity for production of both the high- and low-end products. In reality, the incumbent OEM can make the signal credible by ways such as outsourcing the production of low-end versions that do not include proprietary technology, or showing the supplier that the high-end version requires some special components that are not needed for the low-end and thus the produced quantity of high-end products is limited by these special components. The incumbent OM may gain leverage with the supplier for a lower wholesale price by mimicking the effect of low-end competition from a rival OEM.

On the other hand, a commitment to a limited capacity may in itself invite the entry of a rival OEM because the rival now faces a lower wholesale price as well. Thus, the incumbent's capacity pre-commitment has strategic effects on both the supplier who decides whether or not to offer a lower wholesale price, and the rival OEM who determines whether or not to produce low-end products and enter the market to take advantage of a possible lower wholesale price.

To study those questions, we use a quantity competition model to present the strategic issues involved among an incumbent original equipment manufacturer (OEM) and a potential entrant producing a low end version of the product facing a common supplier. To illustrate the results, we consider a simple supply chain consisting of a component supplier (S), an incumbent OEM (I), and a potential entrant (E). The product under consideration has two versions: the high end version and the low end version. For simplicity, we refer to the two products as product H and product L, respectively. We assume that the perceived quality level of the low end version product L is lower than product H. To produce both products H and L, firm I and firm E need to procure certain components from the common supplier S. We assume the supplier incurs a unit production cost for the component. To produce the high end product H, a unit premium is incurred. We assume the production cost for product L is zero. This assumption is not a requirement in our model. It is made for the simplicity of notation and model analysis. Furthermore, we assume that the incumbent can produce both product H and L. But the entrant can only produce product L. The inefficiency of the product line is operationalized by assuming that the incumbent OEM can credibly commit to a production capacity (or quantity) for the high end product H.

This work is closely related to three streams of research: market segmentation and product line decisions, distribution channel structure, and capacity decisions. Our model borrows extensively from the market segmentation literature. In their seminal work, Mussa and Rosen (1978, [34]) explore optimal prices and qualities for a product line that is sold to heterogeneous consumers with continuous preferences. Following that, Moorthy (1984, [31]) considers how to determine product prices for a product line with a finite number of consumer classes. Moorthy and Png (1992, [32]) study a two-period model with two different quality products and explore whether or not to introduce both quality products at the first period or only the higher in the first period. Desai (2001, [16]) compares a firm's pricing and quality decisions in a monopoly setting and a duopoly situation and finds that competition can affect a firm's optimal decisions. More recently, Lacourbe et al. (2009, [37]) consider product line decisions with two dimensions of consumer preferences: vertical differentiation for product performance and horizontal differentiation for product feature.

The literature of distribution channel structure is extensive and various issues have been considered (e.g., contract types that can coordinate vertical members in a channel (Jeuland and Shugan 1983, [23]), equilibrium channel structures when there are horizontal competitions (McGuire and Staelin 1983, [28]), the influence of channel power on firms' decisions (Choi 1991, [10], and Lee and Staelin 1997, [26], etc.). Our paper is more related to how the interactions among different firms in a channel affect individual firm's performance. Along this line, Arya et al. (2007, [3]) show that a supplier's encroachment can benefit the retailer as the supplier will lower its wholesale price to support its retailer's demand. When network effects exist, Conner (1995, [12]) demonstrates that an innovator can earn higher profit by encouraging clones from competitors. Sun et al. (2004, [45]) consider an innovator's four strategies: 1) a single-product-monopoly strategy, 2) a technology-licensing strategy, 3) a product-line-extension strategy and 4) a combination strategy of both product line extension and technology licensing. They show that product line extension is superior to licensing when the network effect is weak. Similar to the study of Arya et al. (2007, [3]), we don't consider network externality effects. Though wholesale price is the main driver of firm performance as a result of supplier-manufacturer interactions, this paper considers a different setting and different research questions from Arya et al. (2007, [3]). Planning for capacity is critical for firms as normally a large amount of capital needs to be committed. The study of capacity related issues has been various. Among these include capacity expansion by determining the capacity size, time, location, etc. so as to meet increasing demand over time (Luss 1982, [27]); capacity (equipment) replacement by taking into account future changes in capacity requirement (Rajagopalan et al. 1998, [39]); flexible capacity amount to meet uncertain demand for multiple products (van Mieghem 1998, [49]); simultaneous capacity and production planning to maximize profits over multiple periods (Angelus and Porteus 2002, [2]), etc. Readers interested more in this area please refer to the comprehensive review presented by van Mieghem (2003, [50]). Differing from the above papers, we consider a manufacturer's capacity commitment on its high end product and the impact of this commitment on production decisions of its competitor.

The rest of this chapter is organized as follows. In section 4.2, we describe the key elements of our models. The key results of the models are presented in Sections 4.3 and 4.4. In Section 4.3, we present the results for the cases without strategic suppliers (those models whose names begin with "N"). In Section 4.4, we analyze the models with strategic suppliers and compare them to those without strategic suppliers. Finally, we conclude this chapter by summarizing the key findings and point to some future research directions.

## 4.2 The Basic Model

Consider a supply chain consisting of a supplier (S), an incumbent OEM (I), and a potential entrant OEM (E). We adopt the convention of using feminine pronouns for the OEMs and masculine pronouns for the supplier. In addition, we will henceforth refer to the supplier of the critical component as *the supplier*, the incumbent OEM as *the incumbent*, and the potential entrant OEM as *the entrant*.

The supplier produces a critical component at marginal cost $c$ per unit. The incumbent produces a high-quality product (H) using the component at a cost premium $c_H$. She possibly also produces a low-end version of the product (L). If the entrant enters the market, she produces the low-end version of the product. Without loss of generality, we normalize the marginal cost of producing product L to be zero. Our structural results do not depend on this assumption.

On the demand side, we assume there is a continuum of potential consumers with a total mass of one, each of them buys at most one unit of either product H or product L. Each consumer has a valuation $\theta$ for product quality, which is assumed uniformly distributed on $[0, 1]$. Similarly to Chen et al. (2009, [9]), for any given prices $p_H$ and $p_L$ for product H and L, respectively, the net utility of a consumer with valuation $\theta$ can be stated as follows:

$$u = \begin{cases} \theta - p_H, & \text{if buying product H;} \\ \gamma\,\theta - p_L, & \text{if buying product L;} \\ 0, & \text{if not buying at all.} \end{cases} \tag{4.1}$$

where, $\gamma$ $(0 < \gamma < 1)$ can be interpreted as the relative perceived quality level of product L compared to product H (we normalize the perceived quality level of product H to be one), or the net substitution effect between the two products. We assume that $1 - c_H - c > \frac{\gamma - c}{\gamma} > 0$, that is, product H has a higher potential margin per unit of quality than product L. This assumption implies that $1 - c_H - c > \gamma - c > 0$, which implies that if priced at marginal costs, at least the consumer with highest valuation for quality ($\theta = 1$) is willing to buy both products, although product H is preferred to product L. This assumption rules out the uninteresting case in which product H is never offered by the incumbent, which is derived in Chen et al. (2009, [9]). Notice that this assumption is not as restrictive as it looks like. For example, when $\gamma = 0.8$ (product H and L have comparable perceived quality level), the assumption requires $c_H < 0.25c$. Given the assembly cost for product L is 0, this is at all not an unreasonable assumption.

As is derived in Chen et al. (2009, [9]), when the price for product L is low enough compared to the price for product H, that is, $\gamma p_H > p_L$, both products have positive demand. Consumers with $\theta \in [(p_H - p_L)/(1-\gamma), 1]$ buy H, consumers with $\theta \in [p_L/\gamma, (p_H - p_L)/(1-\gamma)]$ buy L, while consumers with $\theta \in [0, p_L/\gamma]$ buy nothing. Thus, the inverse demand functions for H and L can be derived as follows:

$$p_H(q_H, q_L) = 1 - q_H - \gamma\, q_L; \quad p_L(q_H, q_L) = \gamma(1 - q_H - q_L). \tag{4.2}$$

where, $p_H$ and $p_L$ are the market clearing prices when the quantities are $q_H$ and $q_L$ for product H and product L, respectively.

If only product H is offered, or both products are offered but $\gamma\, p_H < p_L$ (under these prices, any consumers who buy prefer H to L), consumers with $\theta \in [p_H, 1]$ buy product H and the inverse demand function for H is $p_H(q_H, 0) = 1 - q_H$. Similarly, when only product L is offered with price $p_L$, consumers with $\theta \in [p_L/\gamma, 1]$ buy product L and the inverse demand for L is $p_L(0, q_L) = \gamma(1 - q_L)$. Note that as the quality (or substitution) parameter $\gamma$ increases, the inverse demand for L increases at the expense of demand for H. This particular demand model is commonly used in economics literature, among them, Mussa and Rosen (1978, [34]) and Yehezkel (2008, [53]).

As a concluding remark, we solve the case of a vertically integrated channel (a single firm produces and sells both products). In this case, the quantities $q_H$ and $q_L$ are choosen to maximize the channel profit:

$$\pi^{VI}(q_H, q_L) = \begin{cases} [p_H(q_H, q_L) - c_H - c]\, q_H + [p_L(q_H, q_L) - c]\, q_L, & \text{if } q_L > 0; \\ [p_H(q_H, 0) - c_H - c]\, q_H, & \text{otherwise.} \end{cases} \tag{4.3}$$

Thus, the vertical integration quantities are

$$q_H^{VI} = \frac{1}{2}(1 - c_H - c), \quad q_L^{VI} = 0. \tag{4.4}$$

Note that $q_H^{VI} > 0$ from the assumption that $1 - c_H - c > 0$. Intuitively, when the cost-to-quality ratio, $c/\gamma$ of product L is smaller than that of product H ($c_H + c$, recall that the quality level of H is normalized to be one), L is nonetheless *efficient*. Otherwise, L is *inefficient* and the vertically integrated channel will not offer it. We use the same concept of relative efficiency of L compared to H as in Chen et al. (2009, [9]). The difference $(1 - \gamma)\, c/\gamma - c_H$

thus can be interpreted as a measure of the relative efficiency of L to H. Under our assumption, L is inefficient and is not offered by the vertically integated channel, and when $-c_H + (1-\gamma)\, c/\gamma$ increases, L becomes more efficient, as is pointed out in Yehezkel (2008, [53]). In this paper, we focus on the case in which product L is inefficient. We will show that even though product L is inefficient, the entry of an entrant producing product L may nonetheless benefit the incumbent OEM under certain situations.

We use a game theoretical framework to study the interactions among the supplier, the incumbent , and the entrant. We model the problem as a three-stage game. In stage one, the incumbent set a capacity $K$ for product H and incurs a sunk cost $c_H$ per unit capacity installed if she pre-commits on the capacity level. Otherwise, do nothing in this stage. In stage two, the supplier sets a wholesale price $w$ on the components after observing the incumbent's capacity level $K$ for product H if the incumbent pre-commits to it at stage one. In the final stage, the incumbent determines the sales quantities of product H and L. The entrant determines whether to enter the market or not. If she enters, she determines the sales quantity for product L. We use a linear wholesale price contract for the supplier to sell the components to the incumbent and entrant. As pointed out in Arya et al. (2007, [3]), linear wholesale price contracts are commonly adopted in models of channel conflict and also are prevalent in practice. The timeline is depicted in Figure 4.1.

In order to study the effects of product line inefficiency and competition, we compare the cases with and without competition, the cases with and without capacity pre-commitment. For notational simplicity, we use the naming convention "ABC" to name each model we consider in this article, where "A" stands for the situation whether there are strategic suppliers ("S") or not ("N"), "B" stands for whether the incumbent pre-commits on capacity for product H ("C") or not ("N"), and "C" stands for whether the entrant enters the market ("R") or not ("N"). For example, model "NNN" stands for the case in which there is no strategic suppliers, no capacity pre-commitment, and no competition[1]. The features of each of the seven different models considered in this work are summarized in Table 4.1 and the relationship among the seven possible combinations are provided in Figure **??**[2].

## 4.3 The Benchmark: Without Strategic Suppliers

As a benchmark, we first study the models without strategic suppliers. Three models are considered in this section: a) Model NNN: in this setting, there is no strategic supplier, no capacity commitment, and no competition; b) Model NNR: this is the model with competition; and c) Model NCR: the model with capacity commitment and competition. Note that model NNN is the same as the vertically integrated case and model NCN is equivalent to model NNN.

In model NNR, the incumbent and the rival compete in the same mar-

[1]This is the case of a monopolist producing the products with unlimited capacity, or equivalently, the vertically integrated case.

[2]The combination "NCN" is the same as "NNN" under our setting.

ket: the incumbent sets output quantities for both products, while the rival decides whether to enter the market and how much product L to produce if she enters. The results are presented in the following lemma.

**Lemma 4.3.1.** ***(Equilibrium of Model NNR)*** *In the presence of competition from the entrant, when there is no strategic supplier, the equilibrium can be characterized as follows:*

1. *When* $c_H \in \left[0, \max\left\{0, \frac{(2-\gamma)c-\gamma}{\gamma}\right\}\right]$, *the incumbent sets* $q_H^{NNR} = \frac{1-c-c_H}{2}$ *and* $q_{IL}^{NNR} = 0$, *and the entrant does not enter.*

2. *When* $c_H \in \left[\max\left\{0, \frac{(2-\gamma)c-\gamma}{\gamma}\right\}, \frac{c(1-\gamma)}{\gamma}\right]$, *the incumbent sets* $q_H^{NNR} = \frac{2-\gamma-c-2c_H}{4-\gamma}$ *and* $q_{IL}^{NNR} = 0$, *and the entrant enters and sets* $q_{EL}^{NNR} = \frac{(1+c+c_H)\gamma-2c}{\gamma(1-\gamma)}$.

All proofs in the paper are provided in appendix. The regions are shown in Figure 4.3b. It is well-known that competition hurts the monopolist with non-strategic supplier. The following result shows that the result is true even if the competition is resulted from an entrant producing a low end version of the product. In this case, demand cannibalization is the only effect of competition, which lowers the incumbent's profit.

**Proposition 4.3.2.** ***(Effect of Competition without Strategic Supplier)*** *Without strategic supplier, the presence of low end competition hurts the incumbent: the incumbent's sales quantities for both products and profit are all lower than when there is no competition.*

The results are presented in Figure 4.4. In Figure 4.4a, we plot the sales quantities for both products in model NNN and NNR. The solid line indicates the incumbent's sales quantities for product H in both models. The top line is the sales quantity in model NNN, the bottom line is in model NNR. Similarly, the dashed lines are for sales quantities for product L and the top line is in model NNN and the bottom one is in model NNR. The dotted line is for the rival's sales quantitiy for product L in model NNR. With the presence of a potential entrant in the market, the incumbent's sales quantities for both products decrease: there is demand cannibalization from the entry of the rival. Figure 4.4b shows that the incumbent's profit is lower with an entrant than without.

Now consider the case in which the incumbent pre-commits to a capacity level $K$ for the high end product at a unit capacity cost $c_H$. That is, the incumbent can only produce the low end products (or produce nothing) if there are more than $K$ units of critical components available. In this setting, the incumbent first pre-commits to the capacity level $K$ for the high end product. Then the incumbent determines the sales quantities $q_H$ and $q_{IL}$ for the high end and low end products, respectively; simultaneously, the rival determines the sales quantity $q_{EL}$ for the low end product.

Given the capacity level $K$ for product H and the rival's sales quantity $q_{EL}$, the incumbent's decisions are the sales quantities $q_H$ and $q_{IL}$ which satisfies $0 \le q_H \le K$ and $0 \le q_{IL}$. The incumbent's problem is given by:

$$\max_{0\le q_H\le K, q_{IL}\ge 0} \left[p_H\left(q_H, q_{IL}+q_{EL}\right)-c\right]q_H+\left[p_L\left(q_H, q_{IL}+q_{EL}\right)-c\right]q_{IL}-c_H\cdot K. \tag{4.5}$$

Similarly, the rival sets her sales quantity $q_{EL}$ for product L to maximize her profit as long as the profit is nonnegative:

$$\max_{q_{EL}\ge 0} \left[p_L\left(q_H, q_{IL}+q_{EL}\right)-c\right]q_{EL}. \tag{4.6}$$

The equilibrium, including the optimal capacity level $K$ for product H, the incumbent's sales quantities $q_H$ and $q_{IL}$, and the entrant's sales quantity $q_{EL}$ ($q_{EL}=0$ stands for the case in which the entrant does not enter the market) are summarized in the following lemma.

**Lemma 4.3.3.** ***(Equilibrium of Model NCR)*** *Without strategic supplier, if the supplier pre-commits on capacity for product H, then there exist four mutually exclusive regions $R^i_{NCR}$ for the values of $(c,\, c_H)$, $i=1,2,3,4$, such that the equilibrium can be characterized as follows:*

1. *When $(c,\, c_H)\in R^1_{NCR}$, the incumbent sets $K^{NCR}=\frac{2-\gamma-c}{4-\gamma}$, $q_H^{NCR}=K$, and $q_{IL}^{NCR}=0$, and the entrant enters the market and sets $q_{EL}^{NCR}=\frac{\gamma-c(2-\gamma)}{\gamma(4-\gamma)}$.*

2. *When $(c,\, c_H)\in R^2_{NCR}$, the incumbent sets $K^{NCR}=\frac{2-\gamma-c-2c_H}{4-2\gamma}$, $q_H^{NCR}=K$, and $q_{IL}^{NCR}=0$, and the entrant enters the market and sets $q_{EL}^{NCR}=\frac{1}{2}\left(\frac{\gamma-c}{\gamma}-K\right)$.*

3. *When* $(c, c_H) \in R^3_{NCR}$*, the incumbent sets* $K^{NCR} = \frac{1-c-c_H}{2}$*,* $q_H^{NCR} = K$*, and* $q_{IL}^{NCR} = 0$*, and the entrant does not enter the market.*

4. *When* $(c, c_H) \in R^4_{NCR}$*, the incumbent sets* $K^{NCR} = \frac{\gamma-c}{\gamma}$*,* $q_H^{NCR} = K$*, and* $q_{IL}^{NCR} = 0$*, and the entrant does not enter the market.*

The definition of the four regions are provided in the appendix.

Without strategic supplier, competition plays a sole strategic role in the interaction between the incumbent and the rival: demand cannibalization. Thus, if the incumbent is endowed with the capability of pre-committing on a capacity level for the high end product, he will pre-commit to a higher capacity level than what he would produce without capacity pre-commitment in order to deter potential entry. In this case, by sunking the premium to produce the high end product, the incumbent can credibly signal to the rival a lower cost-to-quality ratio (since the incumbent and the rival will have the same production cost $c$ to produce the high end product and low end product, respectively). This credible signal can help the incumbent to deter entry from the rival and thus achieve higher profit. The above analysis is formalized in the following proposition.

**Proposition 4.3.4.** ***(Effect of Capacity Commitment without Strategic Supplier)*** *Without strategic supplier, if there is potential low end competition from the entrant, the incumbent always pre-commit on a higher capacity level for product H than what he would produce without capacity pre-*

*commitment and does not produce product L. In both cases (with or without capacity pre-commitment), the incumbent is worse off under competition.*

Proposition 4.3.4 tells us that competition always hurts the incumbent without strategic supplier, even when the incumbent is endowed with the capability of pre-committing on a capacity level for the high end product to help to deter potential entry from the rival. In the later sections, we will compare the results to those with strategic suppliers to see how the presence of strategic supplier changes the strategic effects of both capacity commitment and competition.

## 4.4 Analysis and Findings with Strategic Suppliers

In this section, we consider how the presence of a strategic supplier changes the effects of competition and capacity pre-commitment. More specifically, we study how the presence of a strategic supplier changes the role of competition by comparing the models without (model SNN) and with competition (model SNR). Next, we study the possibility of a monopoly OEM mimicing the strategic effect of low end competition by limiting her capacity for product H without demand cannibalization in Section 4.4.2. We compare the model with capacity precommitment (SCN) to model SNN. Finally, in Section 4.4.3, we consider the case in which the incumbent faces low end competition and she can pre-commit on a capacity for product H at the same time (model SCR, which will be compared to model SCN). We focus our attention on how should the incumbent balance the strategic issues related to the supplier and the competitor.

### 4.4.1 Effect of Competition with Strategic Supplier

In this section, we study the effect of competition with strategic supplier when the incumbent is not endowed with the capability of pre-committing on capacity level for the high end product (model SNN vs. model SNR). We show that, contrast to the case of non-strategic supplier, low end competition may benefit the incumbent. The reasoning is as follows. In order to induce the rival to produce the inefficient low end version product, the strategic supplier has to lower his wholesale price. Under certain conditions, the benefit from the lower wholesale price can offerset the demand cannibalization from the low end competition for the incumbent. Thus the incumbent may benefit from the low end competition.

In model SNN, the supplier sets the wholesale price $w$ first. After observing the wholesale price, the incumbent determines the sales quantities $q_H$ and $q_{IL}$ for the high end and low end products, respectively. For any given wholesale price $w$, the incumbent produces only the high end product since the low end version is inefficient under our assumption $c_H \leq \frac{c(1-\gamma)}{\gamma}$. Thus, the incumbent's problem is:

$$\max_{q_H \geq 0} \left[p_H\left(q_H, 0\right) - c_H - w\right] q_H$$

and the incumbent produces $q_H\left(w\right) = \left(1 - c_H - w\right)/2$ as long as $w \leq 1 - c_H$; otherwise, the incumbent produces $q_H\left(w\right) = 0$. Thus, the supplier's problem is to find a wholesale price $w$ to solve the following problem:

$$\max_{0\leq w\leq 1-c_H} (w-c)(1-c_H-w)/2.$$

The optimal wholesale price is given by $w^{SNN} = (1+c-c_H)/2$. In equilibrium, the supplier's profit is $\pi_S^{SNN} = (1-c-c_H)^2/8$. The incumbent's sales quantities are $q_H^{SNN} = (1-c-c_H)/4$ and $q_{IL}^{SNN} = 0$, and his profit is $\pi_I^{SNN} = (1-c-c_H)^2/16$.

In model SNR, after observing the wholesale price $w$ from the supplier, the incumbent and the rival simultaneously determine their sales quantities $q_H$, $q_{IL}$, and $q_{EL}$ (rival's sales quantity of the low end product). Given the wholesale price $w$ and rival's sales quantity $q_{EL}$, the incumbent's problem is:

$$\max_{q_H\geq 0, q_{IL}\geq 0} [p_H(q_H, q_{IL}+q_{EL}) - c_H - w]\, q_H + [p_L(q_H, q_{IL}+q_{EL}) - w]\, q_{IL}.$$

The solution for $q_H$ is given by

$$q_H(q_{IL}, q_{EL}, w) = \begin{cases} \frac{1-c_H-w-2\gamma q_{IL}-\gamma q_{EL}}{2}, & if\ q_{EL}+2q_{IL} \leq \frac{1-c_H-w}{\gamma}; \\ 0, & if\ q_{EL}+2q_{IL} \geq \frac{1-c_H-w}{\gamma}. \end{cases}$$

and the solution for $q_{IL}$ is given by

$$q_{IL}(q_H, q_{EL}, w) = \begin{cases} \frac{1}{2}\left(1-q_{EL}-2q_H-\frac{w}{\gamma}\right), & if\ q_{EL}+2q_H \leq \frac{\gamma-w}{\gamma}; \\ 0, & if\ q_{EL}+2q_H \geq \frac{\gamma-w}{\gamma}. \end{cases}$$

Similarly, the entrant solves her problem by finding the optimal value of $q_{EL}$ given $q_H$ and $q_{IL}$:

$$\max_{q_{EL}\geq 0} [p_L(q_H, q_{IL}+q_{EL}) - w]\, q_{EL}.$$

The solution is given by

$$q_{EL}\left(q_H, q_{IL}, w\right) = \begin{cases} \frac{\gamma(1-q_H-q_{IL})-w}{2\gamma}, & if\ q_H + q_{IL} \leq \frac{\gamma-w}{\gamma}; \\ 0, & if\ q_H + q_{IL} \geq \frac{\gamma-w}{\gamma}. \end{cases}$$

Combining the solutions for the incumbent and the entrant, we have the quantity decisions for both the incumbent and the entrant as shown in Table 4.2.

The supplier determines the wholesale price $w$ to maximize his profit given the incumbent and the rival's best response functions. The supplier's problem is:

$$\max_{w \geq c} \left(w - c\right) \left[q_H\left(w\right) + q_{IL}\left(w\right) + q_{EL}\left(w\right)\right], \tag{4.7}$$

where, $q_H\left(w\right)$, $q_{IL}\left(w\right)$, and $q_{EL}\left(w\right)$ are as given in Table 4.2. The equilibrium is summarized in the following lemma.

**Lemma 4.4.1.** ***(Equilibrium of Model SNR)*** *In model SNR, there exist two mutually exclusive regions $R^1_{SNR}$ and $R^2_{SNR}$ for the values of $(\gamma, c, c_H)$, such that the equilibrium can be characterized as follows:*

1. *When $(\gamma, c, c_H) \in R^1_{SNR}$, the supplier sets the wholesale price $w = \frac{\gamma(3-\gamma-c_H)+2c}{4}$, the incumbent orders $q_H = \frac{8-2c-(8-\gamma)c_H-\gamma(7-\gamma)}{\gamma(4-\gamma)}$, $q_{IL} = 0$, and the rival orders $q_{RL} = \frac{\gamma((5-\gamma)\gamma+(6-\gamma)c_H-2)-2c(2-\gamma)}{4\gamma(4-\gamma)}$;*

2. *When $(\gamma, c, c_H) \in R^2_{SNR}$, the supplier sets the wholesale price $w = \frac{1+c-c_H}{2}$, the incumbent orders $q_H = \frac{1-c-c_H}{4}$, $q_{IL} = 0$, and the rival orders $q_{RL} = 0$.*

The two regions are defined in the appendix. An interesting question is that if the incumbent is always not better off when the rival orders positive quantity from the supplier. That is, does low end competition always hurt the incumbent? The answer is no.

**Proposition 4.4.2.** ***(Effect of Low End Competition with Strategic Suppliers)*** *In the presence of a strategic supplier, there exists a region $R_{SNR}$ for the values of $(\gamma, c, c_H)$, such that the incumbent may benefit from low end competition from the entrant if and only if $(\gamma, c, c_H) \in R_{SNR}$. Furthermore, the supplier charges a lower wholesale price when there is low end competition than when there is no competition.*

The underlying reason of the benefit for the incumbent from the low end competition is as follows. The incumbent would not produce the inefficient low end version product. The rival OEM produces and sells the inefficient low end version product. In order to bring the rival to the market, the only way for the supplier is to lower his wholesale price so that the rival can also enjoy a nonnegative profit. At certain range, the benefit for the incumbent from the lower wholesale price offsets the demand cannibalization from the low end competitor.

### 4.4.2 Incumbent's Capacity Pre-Commitment for High End Products

With the presence of strategic supplier, we have shown in the above section that the incumbent can benefit from the low end competition introduced by a rival OEM as long as the component cost is not too high and the premium for the high end product is not too high. In this section, we will answer the question that if the incumbent can obtain similar strategic effect without demand cannibalization. In this paper, we allow the incumbent to pre-commit on the capacity for the high end version product before he observes the wholesale price set by the supplier. We will show that the incumbent will obtain similar strategic effect as low end competition but without cannibalization. We also show that the incumbent will always pre-commit to a lower capacity level for the high end product that what he would produce without capacity pre-commitment.

The sequence of the decisions is specified as follows: First, the incumbent decides on the capacity level $K$ for the high end product; Second, the supplier observes the capacity level $K$ and sets his wholesale price $w$. Finally, the incumbent sets his order quantities $q_H$ and $q_{IL}$ for the high end and low end products, respectively. As usual, we use backward induction to solve this problem. Notice that this is model SCN.

At the third stage, the incumbent's problem is:

$$\max_{0 \leq q_H \leq K, q_{IL} \geq 0} \left[ p_H \left( q_H, q_{IL} \right) - w \right] q_H + \left[ p_L \left( q_H, q_{IL} \right) - w \right] q_{IL} - c_H K.$$

Using Lagrangian method, we can find the incumbent's optimal decisions $q_H$ and $q_{IL}$ given the wholesale price $w$ and the capacity level $K$:

$$\begin{cases} q_H = K, & q_{IL} = \frac{\gamma - w}{2\gamma} - K, & \text{if } 0 \leq w \leq \gamma \left(1 - 2K\right); \\ q_H = K, & q_{IL} = 0, & \text{if } \gamma \left(1 - 2K\right) \leq w \leq 1 - 2K; \\ q_H = \frac{1 - w}{2}, & q_{IL} = 0, & \text{if } w \geq 1 - 2K. \end{cases}$$

In the second stage, the supplier's profit is thus given by

$$\begin{cases} \frac{1}{2\gamma}\left(w-c\right)\left(\gamma-w\right), & \text{if } 0 \le w \le \gamma\left(1-2K\right); \\ K\left(w-c\right), & \text{if } \gamma\left(1-2K\right) \le w \le 1-2K; \\ \frac{1}{2}\left(w-c\right)\left(1-w\right), & \text{if } w \ge 1-2K. \end{cases} \tag{4.8}$$

Given the capacity level $K$ for product H, the supplier's optimal choice of wholesale price $w$ is characterized in the following result.

**Lemma 4.4.3.** ***(Supplier's Optimal Wholesale Price in Model SCN)*** *Given the capacity level $K$ for product H, the supplier's optimal wholesale price $w$ is given as follows:*

1. *When $0 \le K \le \frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}}$, the supplier sets $w = \frac{c+\gamma}{2}$; the incumbent produces product H up to capacity $K$ and produce some product L;*

2. *When $\frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}} \le K \le \frac{1-c}{4}$, the supplier sets $w = 1-2K$; the incumbent produces product H up to capacity $K$ and does not produce any product L;*

3. *When $K \ge \frac{1-c}{4}$, the supplier sets $w = \frac{1+c}{2}$; the incumbent produces product H up to $\frac{1-c}{4}$ and does not produce product L.*

For small values of capacity level $K$, since the total units of product H and L are constant when $K$ increases, the supplier is not willing to change the wholesale price. However, when capacity level $K$ takes intermediate values, when the incumbent increases $K$, the supplier can sell more units of the component. Thus, the supplier is willing to lower the wholesale price. Finally, when $K$ is large enough, increasing $K$ has no effect on incumbent's ordering quantity for the component. The supplier keeps the wholesale price a constant. Based on the supplier's optimal response for any given value of the capacity level $K$ for product H, we can identify the incumbent's optimal choice of the capacity level $K$. We summarize the result in the following lemma.

**Lemma 4.4.4.** ***(Equilibrium of Model SCN)*** *When the incumbent pre-commit to a capacity level $K$ for product H, there exists a threshold value $c_H^{SCN}$ such that $0 \leq c_H^{SCN} < \frac{c-c\gamma}{\gamma}$ and the equilibrium of the 3-stage game can be characterized as follows:*

1. *When $0 \leq c_H \leq c_H^{SCN}$, the incumbent sets a capacity level $K = \frac{1-c}{4}$; the supplier charges a wholesale price $w = \frac{1+c}{2}$; the incumbent produces product H up to capacity $K$ and does not produce product L.*

2. *When $c_H^{SCN} \leq c_H < \frac{c-c\gamma}{\gamma}$, the incumbent sets a capacity level $K = \frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}}$; the supplier charges a wholesale price $w = \frac{\gamma+c}{2}$; the incumbent produces product H up to capacity $K$ and does not produce product L.*

Recall that when there is no strategic supplier, the incumbent always pre-commit to a higher capacity for product H. This result tells us that when the incumbent has to pre-commit to a capacity level for product H before observing the wholesale price for the component, the incumbent intents to commit to a lower capacity level, which induces the supplier to charge a lower wholesale price. However, the pre-commitment on capacity level for product H may or may not benefit the incumbent. The effect of capacity pre-commitment is two-folded: on one hand, pre-committing to a lower capacity for product H helps to induce a lower wholesale price; on the other hand, pre-committing to a lower capacity for product H limits the sales quantity of product H. The following results formalize the above statement.

**Proposition 4.4.5.** ***(Effect of Capacity Pre-Commitment with Strategic Suppliers)*** *In model SCN, there exists a threshold value $c_{H,1}^{SCN}$ such that $c_H^{SCN} \leq c_{H,1}^{SCN} \leq \frac{c-c\gamma}{\gamma}$ and the incumbent is worse off by pre-commiting on a capacity for product H when $0 \leq c_H \leq c_{H,1}^{SCN}$. However, the incumbent obtains higher profit by pre-commiting on a capacity for product H when $c_{H,1}^{SCN} < c_H < \frac{c-c\gamma}{\gamma}$.*

When the production premium for product H is relatively large, although the reduction on wholesale price is small, the reduced sales quantity by limiting the capacity for product H is also small. However, the rate of reduction on wholesale price is higher (two times) than that on sales quantity. Thus, when the production premium is large enough, the former effect dominates and the incumbent benefits from this limited capacity on product H.

### 4.4.3 Strategic Issues Related to Supplier and Competitor

Now we investigate how should the incumbent balance the two strategic issues related to supplier and competitor: in the presence of a strategic supplier, the incumbent intends to pre-commit to a lower capacity for product H to induce lower wholesale price and to pre-commit to a higher capacity level to help to deter entry when there is no strategic suppliers. This case is denoted by model SCR. The sequence of events in this three stage game is as follows: In stage one, the incumbent sets a capacity $K$ for product H at a unit capacity cost $c_H$; in stage two, the supplier sets a wholesale price $w$ for the component after observing the capacity $K$ for product H; in stage three, the incumbent and the entrant determine their sales quantities for both products (the entrant only determines the sales quantity for product L if she enters the market) simultaneously.

## 4.5 Conclusion

In many industries, OEMs must obtain critical components from a few powerful suppliers. For example, OEMs that produce information technology hardware typically interact with highly concentrated supply industries that are dominated by a few key participants, e.g. Microsoft, Intel, etc. To the extent that the OEMs are also concentrated, e.g. Dell, Hewlett-Packard, etc., the interactions between the suppliers of critical components and the OEMs are *strategic*. In order to better understand how an OEM should interact with a strategic supplier, we consider how these interactions are influenced by the structure of the market that is served by the OEM. We first demonstrate that the presence of a rival who produces a low-end substitute for the OEM's product provides an incentive for the supplier to offer lower wholesale prices. So long as the rival's product is not too close a substitute for that of the OEM, she benefits more from the lower wholesale prices offered by the supplier than she is harmed by the cannibalization of her end demand. We then turn our attention to the question of how the interactions with a strategic supplier influence the OEM's decision on whether to extend her product line by introducing her own low-end substitute. We find that, if she can credibly commit to limiting the amount of capacity that will be available to produce the original version of her product, then interactions with a strategic supplier will tend to cause her to offer a broader product line. This result highlights an important trade-off that is faced an OEM who interacts with strategic suppliers and faces the threat of entry from rivals: By restricting her capacity to produce the original version of her product, the OEM can improve her strategic positioning vis-a-vis the supplier, but such a restriction of capacity will also invite the entry of rivals. The final portion of our work investigates how such an OEM can invest in capacity to balance this trade-off among strategic priorities.

We believe this research makes a valuable contribution to the OM literature. This research is closely related to the literature of capacity commitment and product line design with or without competition. The operations management literature on capacity issues typically looks at capacity investment when there are uncertainties such as demand or supply risks. Meanwhile, the marketing literature on product line design has continually put more emphasis on issues of how many different quality products to offer and how to deter entrance of competition. This rese(Note: Liwen, this paper discuss the rationale of using linear prices and cites many papers that consider only linear prices. We should cite these papers too.)arch bridges capacity investment in OM literature with product line design in the marketing literature, and provides managerial insights into the strategic interactions between an incumbent OEM and her supplier as well as a rival OEM.

| Model | Strategic Supplier | Capacity Commitment | Competition |
|---|---|---|---|
| NNN | N | N | N |
| NNR | N | N | Y |
| SNN | Y | N | N |
| SNR | Y | N | Y |
| SCN | Y | Y | N |
| SCR | Y | Y | Y |

Table 4.1: The features of models considered in the paper.

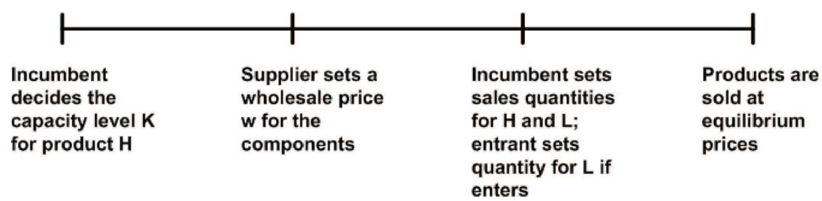



Figure 4.1: Timing of events in base model.

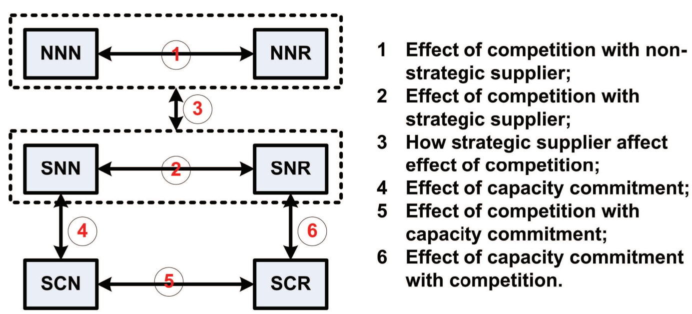
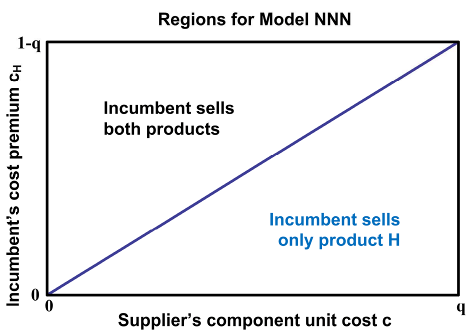
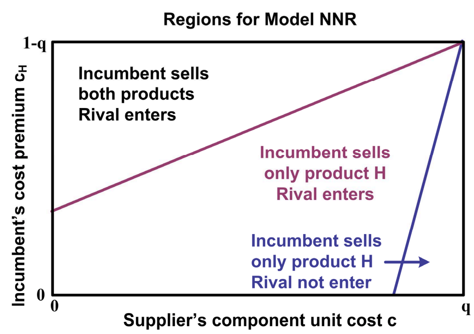


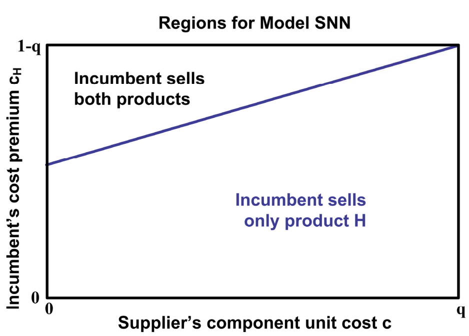

Figure 4.2: The structure of the analysis of the six models in the paper.



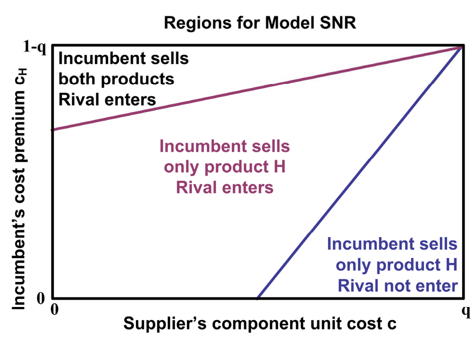

(a) Model NNN




(b) Model NNR

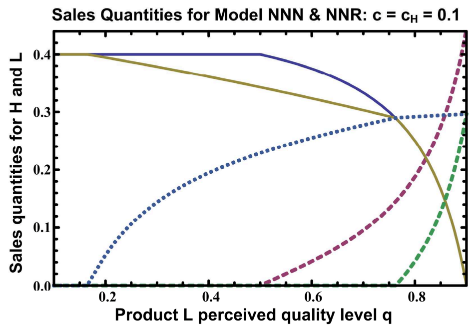



(c) Model SNN

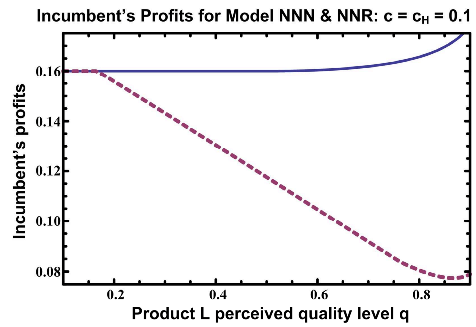



(d) Model SNR

Figure 4.3: The equilibrium market structure for model NNN, NNR, SNN, and SNR.




(a) Sales quantities




(b) Incumbent's profits

Figure 4.4: The sales quantities for product H and L and the incumbent's profits in model NNN and NNR.

| | $R_{1,w}^{SNR}$ | $R_{2,w}^{SNR}$ | $R_{3,w}^{SNR}$ |
|---|---|---|---|
| $q_H(w)$ | $\frac{1-c_H-\gamma}{2(1-\gamma)}$ | $\frac{2-2c_H-\gamma-w}{4-\gamma}$ | $\frac{1-c_H-w}{2}$ |
| $q_{IL}(w)$ | $\frac{c_H}{2-2\gamma}-\frac{\gamma+2w}{6\gamma}$ | $0$ | $0$ |
| $q_{EL}(w)$ | $\frac{\gamma-w}{3\gamma}$ | $\frac{\gamma(1-c_H-w)-2w}{\gamma(4-\gamma)}$ | $0$ |

Table 4.2: Optimal production quantities of both products for any given value of the wholesale price $w$ for the component, where, the regions are given as follows:

$$R_{1,w}^{SNR} = \left\{\frac{\gamma}{3} < c < \gamma,\ \frac{1-\gamma}{3} \le c_H < \frac{c(1-\gamma)}{\gamma},\ w \le \frac{3\gamma c_H - \gamma + \gamma^2}{2-2\gamma}\right\},$$
$$R_{3,w}^{SNR} = \left\{0 < c < \gamma,\ 0 \le c_H < \frac{c(1-\gamma)}{\gamma},\ w \ge \frac{\gamma + \gamma c_H}{2-\gamma}\right\},$$
$$R_{2,w}^{SNR} = \left\{0 < c < \gamma,\ 0 \le c_H < \frac{c(1-\gamma)}{\gamma},\ w \ge 0\right\} \setminus \left(R_{1,w}^{SNR} \cup R_{3,w}^{SNR}\right).$$

# Chapter 5

# Private Label Distribution

## 5.1 Introduction

Private label (also called store brand) products are widely seen in retailscape during the last decades. The world is changing from dominated by manufacturer brands to a mix of manufacturer brands and retailer owned brands. The name "private labels" comes from the fact that most of the retailer brands, if not all, were carried exclusively by the owners of those brands. For example, major U.S. retail chains such as WalMart, Target, JCPenny, and big boxes such as Costco and Sam's Club, have aggressively entered the private label markets during the last decades. Costco's Kirkland Signature, JCPenny's Arizona, and other retailer owned brands become more and more popular among consumers now.

However, over time, we started to notice a change in the retail industry. A few retailers began to distribute their so-called private labels through their competing retailers. For example, starting on the fall of 2008, Safeway began to roll out its popular O' Organics organic foods and Eating Right healthy foods store brands to a wider audience – competing food retailers in the U.S. – along with to grocers globally. At the end of year 2008, as one of the largest office supply providers in U.S, OfficeMax partnered with Safeway to provide office products and school supplies to grocery stores. ...Product selections will include OfficeMax private label products such as award-winning TUL writing instruments and various grades of OfficeMax-brand papers. More recently, Sears Holdings Corp. has agreed to sell its popular Craftsman tool brand through Ace Hardware stores, as the company turns again to outsiders to help grow its sales. Sears roughly 900 department stores will remain the headquarters for sales of Craftsman. In this sense, it is more appropriate to call those private label products retailer brand products. We will adopt this term throughout this paper.

The existing literature on private label vs. national brand has focused mainly on the role of retailer brands as a means of obtaining competitive advantages while dealing with national brand manufacturers. They all assume that retailers who own the retailer brand only sell them in their own stores. However, there is no work in the literature which studies the strategic effect of selling retailer brands through competitors to establish strategic position against the national brand manufacturers. To our best knowledge, our research is the first work in the literature to do so.

Particularly, we are interested in the following research questions. First, if you are a marketing manager of Safeway or OfficeMax, when should you keep your retailer brands private? and when should you share your retailer brands with your competing retailers? If you are the marketing manager of the national brand manufacturer, what's the implication for you when your retailer keeps its retailer brand private or shares it with other retailers of yours? Finally, is it to the channel's best interest when a retailer shares its retailer brand with its competitors? We answer these questions in the rest of this chapter.

## 5.2 Model Setup

Consider an upstream manufacturer (M) that produces a high-quality national brand product (N). M sells the product N through two downstream retailers A and B who distribute product N at zero retail costs. In addition to selling product N, retailer A can also sell a lower-quality retailer brand product (P). We normalize the marginal production cost of both products to be zero for the tractability of the model. However, our qualitative results do not change with positive marginal production costs. Retailer A may sell product P through retailer B by incuring a fixed cost $f > 0$. This cost can be the cost of recruiting dedicated sales force, or the cost of establishing and maintaining the relationship with retailer B, or the cost of acquiring extra production capacity for the extra sales through retailer B.

To investigate retailer A's strategic decision of whether selling the retailer brand product P through retailer B, we consider a base model in which retailer A and retailer B operate in their respective exclusive markets (market A and market B, respectively). Each market is consisting of a continuum of potential consumers, each of whom buys at most one unit of the product. The total mass of consumers for market $A$ is $a \geq 0$, while the total mass of consumers for market $B$ is assumed to be of $b \geq 0$. We study the effect of competition in Section 5.4.

Products and consumers are modeled as specified in Chapter 2.

In general, we use superscript to denote the quantities in different markets, while subscript to denote different channel members and different products. For example, we use $q^A_{n,A}$ to denote retailer A's sales quantity for national brand in market A, while $q^B_{p,B}$ to denote retailer B's sales quantity for retailer brand in market B.

In this chapter, we are interested in the problem of whether retailer A should sell the retailer brand product P through the other retailer, B; If it is better off for retailer A selling her retailer brand product through retailer B, what are the underlying reasons. In order to study these questions, we model the problem as a four-stage game. Generally, retailer A needs to procure extra capacity, establish the relationship with retailer B, or invest in dedicated sales forces, in order to establish the distribution channel through retailer B. We believe retailer A's decision of whether to build the distribution channel through retailer B ($e = 1$) or not ($e = 0$) is a relatively long-run decision compared to the pricing decisions of M and herself. Thus, in our game-theoretical model, this decision is made in stage one. The fixed development cost $f$ occurs upon the decision $e = 1$. In stage two, the manufacturer M sets his wholesale price $w_n$ for the national brand. In stage three, if retailer A sells her retailer brand through retailer B, she sets the wholesale price $w_p$ for the retailer brand; otherwise, she does nothing. In our setting, the national brand manufacturer

M operates nationally, while the two retailers operate locally. Thus, we allow retailer A having the flexibility of setting her wholesale price for the retailer brand after observing M's wholesale price of the national brand in our model. In the final stage, retailer A and B determines their respective sales quantities for each product in each market simultaneously, as is adopted by most of the game-theoretical work in literature. We identify the equilibrium of the game using backward induction. Depending on retailer A's decision in stage one, there are two sub-games: $e = 0$ and $e = 1$. In each case we consider in this article, we first solve the equilibrium for the sub-game $e = 0$. Then, we identify the equilibrium for the sub-game $e = 1$. Finally, we assemble the two sub-games to obtain the equilibrium for the whole game.

## 5.3 Model Analysis and Findings

In this section, we study the case in which retailer A and retailer B operate in independent markets. That is, there is no direct competition in a common market between the two retailers. The only link between them is through the common national brand product supplier M. In Section 5.3.1, we first solve the equilibrium for the sub-game $e = 0$, in which retailer A does not establish the distribution channel through retailer B. The equilibrium for the sub-game $e = 1$ is identified in Section 5.3.2, where retailer A establishes the distribution channel through retailer B. In Section 5.3.4, we obtain the equilibrium for the whole game. To simplify the notation, we normalize the market size of market A to be $a = 1$.

### 5.3.1 3.1 Keeping Retailer Brand Private

In this sub-game, the supplier M charges a wholesale price $w_n$ for the national brand to both retailers. After observing the wholesale price, retailer A decides the sales quantities for both the national brand and the retailer brand; while retailer B sets the sales quantity for the national brand simultaneously. We solve the sub-game using backward induction. Given the wholesale price $w_n$ and retailer B's sales quantity $q_{n,B}^B$ for product N, retailer A sets her quantities $q_{n,A}^A$ and $q_{p,A}^A$ for product N and P, respectively, to maximize her profit:

$$\max_{q_{n,A}^A,\, q_{p,A}^A} \pi_A = \left[p_n\left(q_{p,A}^A, q_{n,A}^A; 1\right) - w_n\right] q_{n,A}^A + p_p\left(q_{p,A}^A, q_{n,A}^A; 1\right) q_{p,A}^A. \tag{5.1}$$

Retailer A's best response is given by

$$q_{n,A}^A = \begin{cases} \frac{1-\gamma-w_n}{2(1-\gamma)}, & \text{if } w_n \le 1-\gamma; \\ 0, & \text{if } w_n \ge 1-\gamma. \end{cases} \qquad q_{p,A}^A = \begin{cases} \frac{w_n}{2(1-\gamma)}, & \text{if } w_n \le 1-\gamma; \\ \frac{1}{2}, & \text{if } w_n \ge 1-\gamma. \end{cases} \tag{5.2}$$

Similarly, for any given values of $w_n$, $q_{n,A}^A$, and $q_{p,A}^A$, retailer B sets the sales quantity $q_{n,B}^B$ for the national brand to maximize her profit:

$$\max_{q_{n,B}^B} \pi_B = \left[p_n\left(0, q_{n,B}^B; b\right) - w_n\right] q_{n,B}^B. \tag{5.3}$$

The best response thus is given by

$$q_{n,B}^B = \frac{1}{2} b\left(1 - w_n\right). \tag{5.4}$$

Notice that the quantity decisions of retailer A and B are independent. That is, they are not functions of the other retailer's quantity decisions. However, they affect the decisions of each other indirectly through the choice of the wholesale price $w_n$ by the common national brand supplier M.

M's problem is to set a wholesale price $w_n$ to maximize the profit:

$$\max_{w_n} \Pi = w_n \left(q_{n,A}^A + q_{n,B}^B\right).$$

Substituting the values of $q_{n,A}^A$ and $q_{n,B}^B$ given in equations (5.2) and (5.4), we have the following:

$$\Pi = \begin{cases} \frac{1}{2} w_n \left(1 + b - bw_H - \frac{w_n}{1-\gamma}\right), & w_n \leq 1 - \gamma; \\ \frac{1}{2} bw_H \left(1 - w_n\right), & w_n \geq 1 - \gamma. \end{cases} \tag{5.5}$$

Thus, the optimal wholesale price is given by

$$w_n = \begin{cases} \frac{(1+b)(1-\gamma)}{2+2b(1-\gamma)}, & \text{if } 0 < \gamma < \hat{\gamma}_0; \\ \frac{1}{2}, & \text{if } \hat{\gamma}_0 \leq \gamma < 1. \end{cases} \tag{5.6}$$

where, $\hat{\gamma}_0 = \frac{1+b}{1+2b}$.

The equilibrium prices, quantities, and profits are provided in Table 5.2.

When the retailer brand is kept private by retailer A, there exists a quality level $\hat{\gamma}_0 = \frac{1+b}{1+2b}$ for the retailer brand, such that when the quality level $\gamma$ of the retailer brand is below the threshold, the manufacturer of the national brand set his wholesale price low enough to induce retailer A selling both her own retailer brand and the national brand. When $\gamma$ exceeds the threshold, the manufacturer deals exclusively with retailer B and charges the monopoly wholesale price.

When retailer A sells both the national brand and her own retailer brand, the equilibrium wholesale price is decreasing in the quality level of the retailer brand: retailer A uses the retailer brand as a leverage against the national brand supplier so as to obtain a better wholesale price. However, when the quality level of the retailer brand is high enough, retailer A has no incentive to sell the national brand which cannibalizes the demand for her own retailer brand. Thus, the national brand only has business with retailer B and he charges the monopoly wholesale price for the national brand.

From the manufacturer's point of view, the independent market B provides a leverage against retailer A who sells her own retailer brand along with the national brand. The larger the size of market B, the more power the manufacturer has against retailer A. Thus, as the market size of market B increases, the manufacturer charges a higher wholesale price. At the same time, the manufacturer is less patient to the price pressure from the retailer brand sold by retailer A. That is, the manufacturer switch to deal exclusively with retailer B for smaller quality level of the retailer brand.

Figure 5.1 demonstrate the equilibrium wholesale price $w_n$ plotted against the quality level of the retailer brand for three different values of the market size of market B.

The above discussion is formally summarized in the following result.

**Lemma 5.3.1.** *Let* $\hat{\gamma}_0 = \frac{1+b}{1+2b}$*. The equilibrium wholesale price* $w_n$ *in the sub-game* $e = 0$ *has the following properties:*

1. $w_n$ *is decreasing in* $\gamma$ *when* $0 < \gamma < \hat{\gamma}_0$*, and is equal to* $\frac{1}{2}$ *when* $\hat{\gamma}_0 \leq \gamma < 1$*;*

2. $w_n$ *is increasing in* $b$ *when* $0 < \gamma < \hat{\gamma}_0$*;*

*3. The threshold value $\hat{\gamma}_0$ is decreasing in $b$.*

When there is no direct connection between the two retailers, retailer B serves as a leverage for the manufacturer of the national brand agains retailer A as discussed above. Thus, as expected, the manufacturer's profit is increasing in the market size $b$ of market B. Similarly, retailer B's profit increases in her own market size. The large market size of market B obviously hurts retailer A when she sells the national brand. It is intuitive for the observation that the manufacturer's profit is decreasing in the quality level $\gamma$ of the retailer brand while retailer A's profit is increasing. However, it is interesting to observe that retailer B's profit is also increasing in $\gamma$. This is due to the fact that when $\gamma$ increases, the manufacturer's wholesale price for the national brand decreases. Thus, retailer B benefits from the reduction of the wholesale price. The sales quantities behave as we anticipated. The above discussion about the comparative statics of the equilibrium is formally presented in Table 5.2.

Next, we consider the case in which retailer A sells her retailer brand through retailer B.

### 5.3.2 Selling Retailer Brand

Now, we consider the sub-game $e = 1$. In this case, after observing the wholesale price $w_n$ for the national brand, retailer A sets a wholesale price $w_p$ for the retailer brand. For given values of $w_n$ and $w_p$, retailer A decides the sales quantities $q_{n,A}^A$ and $q_{p,A}^A$ for the national and retailer brands, respectively, to maximize her profit:

$$\max_{q^A_{n,A},\, q^A_{p,A}} \pi_A = \left[p_n\left(q^A_{p,A}, q^A_{n,A}; 1\right) - w_n\right] q^A_{n,A} + p_p\left(q^A_{p,A}, q^A_{n,A}; 1\right) q^A_{p,A} + w_p q^B_{p,B}.$$

The best response is the same as given in equation (5.2).

Now, retailre B needs to determine the quantities $q^B_{n,B}$ and $q^B_{p,B}$ to sell for the national brand and the retailer brand, respectively, to maximize her own profit:

$$\max_{q^B_{n,B},\, q^B_{p,B}} \pi_B = \left[p_n\left(q^B_{p,B}, q^B_{n,B}; b\right) - w_n\right] q^B_{n,B} + \left[p_p\left(q^B_{p,B}, q^B_{n,B}; b\right) - w_p\right] q^B_{p,B}. \quad (5.7)$$

The best response is given by

$$\begin{cases} q^B_{n,B} = 0, & q^B_{p,B} = \frac{b}{2}\left(1 - \frac{w_p}{\gamma}\right), \quad \text{if } w_n - w_p \geq 1 - \gamma; \\ q^B_{n,B} = \frac{b}{2}\left(1 - \frac{w_n - w_p}{1-\gamma}\right), & q^B_{p,B} = \frac{b(qw_H - w_p)}{2q(1-q)}, \quad \text{if } w_n - w_p \leq 1 - \gamma \textit{ and } \gamma w_H \geq w_p; \\ q^B_{n,B} = \frac{b}{2}\left(1 - w_n\right), & q^B_{p,B} = 0, \quad \text{if } w_n - w_p \leq 1 - \gamma \textit{ and } \gamma w_H \leq w_p \end{cases} \quad (5.8)$$

Retailer A sets wholesale price $w_p$ anticipating retailer B and her own quantity decisions to maximize her profit. The optimal wholesale price $w_p$ given $w_n$ is derived as follows:

$$w_p(w_n) = \begin{cases} \frac{\gamma w_n}{2}, & 0 \leq w_n \leq \frac{2(1-\gamma)}{2-\gamma}; \\ w_n - 1 + \gamma, & \frac{2(1-\gamma)}{2-\gamma} \leq w_n \leq \frac{2-\gamma}{2}; \\ \frac{\gamma}{2}, & \frac{2-\gamma}{2} \leq w_n \leq 1. \end{cases} \quad (5.9)$$

The quantities and profits for given values of $w_n$ can be derived by substituting $w_p(w_n)$ to the corresponding function.

In the first stage, M sets the wholesale price $w_n$ for the national brand, anticipating both retailer's quantity decisions and retailer A's wholesale price for the retailer brand, to maximize his profit:

$$\max_{w_n} \Pi = w_n \left( q_{n,A}^A + q_{n,B}^B \right).$$

The optimal value of $w_n$ is given by

$$w_n = \frac{(1+b)(1-\gamma)}{2+b(2-\gamma)}. \tag{5.10}$$

The equilibrium prices, quantities, and profits are provided in Table 5.3.

Since retailer A sells her retailer brand to retailer B along with the national brand and the manufacturer charges the same wholesale price for both retailers, the manufacturer losses the leverage of dealing exclusively with retailer B. Thus, the manufacturer will keep doing business with both retailers as long as he can make a positive profit. Retailer A always sells both the national brand and the retailer brand at equilibrium.

When the quality level $\gamma$ of the retailer brand increases, the manufacturer faces an increasing price competition from the retailer brand. Thus, the manufacturer lows his wholesale price for the national brand. However, when the market size $b$ of market B increases, the sales of the national brand through retailer B increases. The manufacturer faces less price competition from the retailer brand. Therefore, the manufacturer is able to charge a higher wholesale price for the national brand. Similarly, when $b$ increases, retailer A is able to increase the wholesale price without reducing the sales for the retailer brand significantly.

Interestingly, when $\gamma$ increases, retailer A is not always willing to increase the wholesale price for the retailer brand due to the strategic effect of selling the retailer brand through retailer B. When the retailer brand quality level is relatively high, if retailer A charges a high wholesale price, retailer B will charge a high retail price for the retailer brand, which enables the supplier to charge a higher wholesale price for the national brand. Thus, retailer A faces the trade-off between charging a higher wholesale price for the retailer brand but inducing a higher wholesale price for the national brand and charging a lower wholesale price for the retailer brand but enjoys a lower wholesale price for the national brand. When the market size of market B is not too large, the effects are not negligible and the net effect is that retailer A's wholesale price for the retailer brand may be decreasing in the quality level when it is relatively large.

The above discussion is formally presented in the following result.

**Lemma 5.3.2.** *Let* $\hat{\gamma_1} = \frac{1}{b}\left(2 + 2b - \sqrt{2(1+b)(2+b)}\right)$ *and* $\hat{b_1} = \frac{1}{2}\left(3 + \sqrt{17}\right)$. *In sub-game* $e = 1$, *the wholesale prices* $w_n^1$ *and* $w_p^1$ *have the following properties:*

1. *The national brand wholesale price* $w_n^1$ *is decreasing in* $\gamma$ *and increasing in* $b$.

2. *The retailer brand wholesale price* $w_p^1$ *is concave in* $\gamma$ *and increasing in* $b$.

   (a) *When* $0 < b < \hat{b_1}$, $w_p$ *is increasing in* $\gamma$ *if* $0 < \gamma < \hat{\gamma_1}$ *and decreasing if* $\gamma > \hat{\gamma_1}$;

*(b) When $b > \hat{b}_1$, $w_p$ is increasing in $\gamma$.*

The manufacturer's profit behaves similar to the sub-game $e = 0$: it is increasing in $b$ and decreasing in $\gamma$. Similar results are observed for retailer B's profit. The sales quantities also show the same behavior as in the sub-game $e = 0$. Surprisingly, retailer A's profit is concave in both $\gamma$ and $b$ but not always monotone in $\gamma$ and $b$: retailer A's profit is not always increasing in $\gamma$ and $b$ if she chooses to sell her retailer brand through retailer B. This result is somewhat counterintuitive. We explain it as follows. When the market size $b$ of market B is very large, market B is the dominant market. A large propotion of retailer A's profit comes from selling the retailer brand through retailer B. When the quality level $\gamma$ of the retailer brand is close to that of the national brand, the two brands are less differentiated. Although the sales from the retailer brand for retailer A increases, the price competition between the two brands increases as well. This increasing price competition drives down both the manufacturer and retailer A's profit. When $b$ is relatively small (comparable to the market size of market B), this effect is dominated by the increase in retailer brand sales and thus retailer A's profit increases in the quality level.

For a given quality level, there are two effects when the market size $b$ of market B increases. First, it increases retailer A's sales of the retailer brand. This effect increases as $b$ increases. Second, it increases the price competition between the two brands. This effect increases as $b$ increases. The net effect depends on the value of $b$: when $b$ is very small, the latter effect dominates and retailer A's profit decreases in $b$. When $b$ is large, the former effect dominates and retailer A's profit increases in $b$.

This discussion is formally presented in the following result.

**Lemma 5.3.3.** *In sub-game $e = 1$, retailer A's profit has the following properties:*

1. *When $0 < b < 3$, $\pi_A^1$ is increasing in $\gamma$; When $b > 3$, $\pi_A^1$ is increasing in $\gamma$ when $0 < \gamma < \frac{2(b+1)(b+3)}{b(3b+7)}$ and decreasing in $\gamma$ when $\gamma > \frac{2(b+1)(b+3)}{b(3b+7)}$.*
2. *There exists a threshold value $\hat{b_2} > 0$, such that $\pi_A^1$ is decreasing in $b$ when $0 < b < \hat{b_2}$ and increasing in $b$ when $b > \hat{b_2}$.*

### 5.3.3 Effect of Selling Retailer Brand

For the ease of reference, we define the following notation at equilibrium for each of the two sub-games $e = 0$ and $e = 1$. Note that the retailer brand wholesale price and retailer B's sales quantity of the retailer brand only exist in sub-game $e = 1$.

For any given wholesale price $w_n$ for the national brand charged by the manufacturer, we explore how retailer A and retailer B's sales quantities for the national brand will change if retailer A decides to sell the retailer brand to retailer B. When retailer A keeps the retailer brand private, retailer

| Notation | Description |
|---|---|
| $w_n^e$ | National brand wholesale price |
| $w_p^1$ | Retailer brand wholesale price (in sub-game $e = 1$ only) |
| $\Pi_S^e$ | Supplier's profit |
| $\Pi_{S,j}^e$ | Supplier's profit from retailer $j$, where $j = A$, $B$ |
| $\Pi_{SC}^e$ | Supply chain's profit |
| $\pi_j^e$ | Retailer $j$'s profit, where $j = A$, $B$ |
| $\pi_{j,n}^e$ | Retailer $j$'s profit from national brand, where $j = A$, $B$ |
| $\pi_{j,p}^e$ | Retailer $j$'s profit from retailer brand, where $j = A$, $B$ |
| $q_{n,j}^e$ | Retailer $j$'s sales quantity of the national brand, where $j = A$, $B$ |
| $q_{p,j}^e$ | Retailer $j$'s sales quantity of the retailer brand, where $j = A$, $B$ |

Table 5.1: Notation for equilibrium of sub-game $e = 0$ and $e = 1$.

B's consumers only have one choice, the national brand. Thus, the marginal consumer is the one who is indifferent between buying the national brand and not buying at all. To put in another way, the marginal consumer is the one who derives zero utility from consuming the national brand. When retailer A sells the retailer brand to retailer B, the consumers of retailer B have an alternative option: to purchase the retailer brand. When the retail price of the retailer brand is low enough, the marginal consumer can derive a positive utility from consuming the retailer brand. Thus, the retailer brand might cannibalize some of the demand of the national brand. For retailer A, since the wholesale price of the national brand is given, the sales quantity of the national brand does not change when retailer A sells the retailer brand to retailer B. Therefore, the total sales of the national brand decreases after retailer A selling the retailer brand to retailer B. In order to minimize the loss, the manufacturer has to lower his wholesale price. Hence, we anticipate that the equilibrium wholesale price $w_n^1$ in sub-game $e = 1$ is smaller than the equilibrium wholesale price $w_n^0$ in sub-game $e = 0$. This result is formally stated in the following proposition.

**Proposition 5.3.4.** *The supplier charges a lower wholesale price for the national brand when retailer A selling the retailer brand through retailer B than he would when retailer A sells the retailer brand exclusively in her own store. That is, we have $w_n^0 > w_n^1$.*

This result has a very important implication for marketing managers. It demonstrates the strategic effect of a retailer selling her own retailer brand through another retailer: it helps to induce a lower wholesale price from their national brand manufacturer. This result complements the traditional results of competition theory in the sense that it identifies the bright side of competition when the competitors facing a common supplier in the context of vertically differentiated products market. We thus provide a new angle for the marketing / operations managers to design marketing / operational strategies when facing competitions.

When retailer A sells the retailer brand through retailer B, the retailer brand cannibalizes the national brand demand from retailer B as we described in previous discussion. This demand cannibalization induces a lower wholesale price for the national brand from the manufacturer. We thus conclude that the supplier always achieves lower profit if retailer A sells the retailer brand through retailer B. Based on the direct (increasing retailer brand sales) and the indirect (inducing lower wholesale price for the national brand) effects of selling the retailer brand through retailer B, we anticipate that retailer A benefits from doing so. For retailer B, she obtains extra profits from selling the retailer brand, while enjoying the lower wholesale price for the national brand at the same time. We also expect that retailer B benefit from selling the retailer brand.

The following results summarize the effect of selling the retailer brand through retailer B on retailers' and supplier's profits.

**Lemma 5.3.5.** *The equilibrium profits in sub-games $e = 0$ and $e = 1$ have the following properties:*

1. $\Pi_S^0 \geq \Pi_S^1$, $\pi_A^0 \leq \pi_A^1$, $\pi_B^0 \leq \pi_B^1$, *and* $\Pi_{SC}^0 \leq \Pi_{SC}^1$*;*
2. $\pi_{A,p}^0 \leq \pi_{A,p}^1$ *if and only if* $\gamma \leq \frac{(1+b)\left(3+2b-\sqrt{9+4b}\right)}{2b(2+b)}$.

It is noteworthy to point out that retailer A's profit from retailer brand sales may not always higher when selling through retailer B. This observation implies that the extra sales of the retailer brand from retailer B is not the only drive for retailer A's decision on selling her retailer brand through retailer B. In fact, even when selling through retailer B lowers her profit from retailer brand, retailer A is still willing to do so due to the strategic effect of selling the retailer brand through retailer B: it helps to induce lower wholesale price for the national brand from the supplier. The benefit from the lower wholesale price for the national brand can offset the loss of the retailer brand sales.

In sections 5.3.1 and 5.3.2, we observe that the manufacturer's profits are increasing in market size $b$ of market B. Interestingly, we find that the profits increase in different rates in the two sub-games. When retailer A keeps the retailer brand private, retailer B only need to take into account the wholesale price of the national brand when she decides her order quantity from the manufacturer. When market size increases, retailer B is able to sell more units even under a higher wholesale price. Thus, both the wholesale price and the sales quantity of the national brand at retailer B increase. So does the manufacturer's profit. When retailer A sells the retailer brand to retailer B, however, the manufacturer is not able to increase the wholesale price the same amount because of the presence of the price competition from the retailer brand. Therefore, we anticipate the marginal increase is smaller in sub-game $e = 1$ than that in sub-game $e = 0$. The story for the quality level of the retailer brand is different. When the quality level $\gamma$ is small, the equilibrium wholesale price $w_n^1$ decreases slower than $w_n^0$ when we increase $\gamma$; while $w_n^1$ decreases faster than $w_n^0$ when $\gamma$ is large. These results are summarized in the following lemma.

**Lemma 5.3.6.** *Let $\hat{\gamma}_0 = \frac{1+b}{1+2b}$. The equilibrium wholesale prices $w_n^0$ and $w_n^1$ have the following properties:*

1. *When $0 < \gamma < \hat{\gamma}_0$, we have $\frac{\partial w_n^0}{\partial b} > \frac{\partial w_n^1}{\partial b} > 0$; when $\hat{\gamma}_0 < \gamma < 1$, we have $\frac{\partial w_n^0}{\partial b} = 0 < \frac{\partial w_n^1}{\partial b}$.*

2. *When $0 < \gamma < \hat{\gamma}_0$, we have $0 > \frac{\partial w_n^0}{\partial \gamma} > \frac{\partial w_n^1}{\partial \gamma}$; when $\hat{\gamma}_0 < \gamma < 1$, we have*

$\frac{\partial w_n^0}{\partial \gamma} = 0 > \frac{\partial w_n^1}{\partial \gamma}$.

Given the behavior of the retailers and the manufacturer in both subgames, what will be the optimal decision for retailer A to sell the retailer brand through retailer B or not?

### 5.3.4 Equilibrium and Implications

The subgame perfect equilibrium of the whole pricing game describes the solution to retailer A's optimal channel structure problem. Retailer A choose to sell her retailer brand through retailer B if and only if the benefit of doing so exceeds the fix cost $f$ of establishing the distribution channel. Let $\Delta\pi_A \equiv \pi_A^1 - \pi_A^0$ and $\Delta\Pi_{SC} = \Pi_{SC}^1 - \Pi_{SC}^0$ as the benefit from selling the retailer brand through retailer B for retailer A and the supply chain, respectively.

**Proposition 5.3.7.** *For any given quality level $\gamma$ of the retailer brand product, retailer A sells her retailer brand product through retailer B if and only if $f < \Delta\pi_A$ and the supply chain benefits if and only if $f < \Delta\Pi_{SC}$.*

In the case that the improvement in profit exceeds the fix cost $f$ of establishing the distribution channel, retailer A is better off selling the retailer brand through retailer B. We have the similar statement for the whole supply chain. In this section, we will study the properties of the equilibrium for the whole game.

When the fix cost $f$ is given, an interesting question arising is, when does retailer A have more incentive to sell the retailer brand through retailer

B, with low quality retailer brand or high quality? To answer this question, we need to understand the benefit of selling through retailer B. First, selling the retailer brand through retailer B creates extra profit for retailer A. This profit is increasing in the quality level $\gamma$ of the retailer brand. However, the marginal benefit is decreasing in $\gamma$. Second, it induces lower wholesale price for the national brand. Retailer A achieves higher profit from the sales of the national brand. The benefit increases in the quality level of the retailer brand. Finally, selling the retailer brand through retailer B reduces retailer A's profit from the sales of the retailer brand in market A. When the market size of market B is too small, the total benefit of selling the retailer brand through retailer B increases in the quality level of the retailer brand. When the market size of market B is large enough, the benefit first increases, then decreases in the quality level of the retailer brand. The properties are summarized in the following lemma.

**Lemma 5.3.8.** *The function $\Delta\pi_A$ has the following properties:*

1. *It is concave in the quality level $\gamma$ of the retailer brand at both intervals $\left(0, \frac{1+b}{1+2b}\right)$ and $\left(\frac{1+b}{1+2b}, 1\right)$; however, it is not concave in $(0,1)$ in general.*
2. *At the interval $\left(0, \frac{1+b}{1+2b}\right)$, there exists a threshold value $b_A^1 > 0$, such that $\Delta\pi_A$ is increasing in $\gamma$ if and only if $b \geq b_A^1$; when $0 < b < b_A^1$, it is first increasing then decreasing in $\gamma$.*
3. *At the interval $\left(\frac{1+b}{1+2b}, 1\right)$, there exists a threshold value $b_A^2 > 0$, such that $\Delta\pi_A$ is decreasing in $\gamma$ if and only if $b \leq b_A^2$; when $b > b_A^2$, it is first*

*increasing then decreasing in $\gamma$.*

This lemma has an very interesting implication. It says for a not very high fix cost $f$, retailer A only sells her retailer brand through retailer B when the quality level of the retailer brand is neither too low nor too high. The result is intuitive when the quality level of the retailer brand is too low. As mentioned before, there are two effects by selling the retailer brand through retailer B. First, it induces a lower wholesale price of the national brand from the supplier. Second, it creates extra profit from the sales of the retailer brand through retailer B. When the quality level of the retailer brand is too low, the benefit from both effects is not high enough to offset the fix cost. It is somewhat counterintuitive that retailer A does not want to sell the retailer brand through retailer B when the quality level of the retailer brand is too high. At that case, in sub-game $e = 1$, the sales of retailer brand from market B decreases as the quality level of the retailer brand increases; while the wholesale price of the national brand increases. However, in sub-game $e = 0$, the wholesale price of the national brand is constant. Thus, the benefit from selling the retailer brand diminishing as the quality level of the retailer brand increases. We formalize the above discussion in the following proposition.

**Proposition 5.3.9.** *For any given fix cost $f$, there exist four threshold values $\gamma_L^1$, $\gamma_U^1$, $\gamma_L^2$, and $\gamma_U^2$, $0 \leq \gamma_L^1 \leq \gamma_U^1 \leq \frac{1+b}{1+2b} \leq \gamma_L^2 \leq \gamma_U^2 \leq 1$, such that retailer A sells the retailer brand through retailer B if and only if $\gamma_L^1 \leq \gamma \leq \gamma_U^1$ or $\gamma_L^2 \leq \gamma \leq \gamma_U^2$.*

The two intervals may be separated. This is due to the fact that the function $\Delta\pi_A$ is discontinuous at $\gamma = \frac{1+b}{1+2b}$. When the market size of market B is too large, we have $\gamma_U^1 = \gamma_L^2 = \frac{1+b}{1+2b}$.

The function $\Delta\pi_A$ is increasing in the market size $b$ of market B. Thus, we have the following result.

**Proposition 5.3.10.** *For any given fix cost $f$, there exists a threshold value $b_A > 0$ such that retailer A sells the retailer brand through retailer B if and only if $b \geq b_A$.*

The supply chain as a whole may or may not benefit from retailer A selling the retailer brand through retailer B. Define four mutually exclusive regions $I$, $II$, $III$, and $IV$ as follows: a) in regions $I$ and $II$, retailer A does not sell the retailer brand through retailer B, while in regions $III$ and $IV$ she does; b) in regions $I$ and $III$, the supply chain does not benefit from retailer A selling the retailer brand through retailer B, while in regions $II$ and $IV$ it does. The following proposition provides characteristics of the four regions.

**Proposition 5.3.11.** *There exists two threshold values $b_1$ and $b_2$, where $b_1$ is the unique solution of $2b^3 + 3b^2 - 3b - 3$ on $[1, \infty)$ and $b_2 = 3$, such that for any $s \in \left(0, \frac{1+b}{1+2b}\right)$, $\Delta\pi_A$ and $\Delta\Pi_{SC}$ have the following properties:*

1. *When $0 < b < b_1$, we have $\Delta\pi_A > \Delta\Pi_{SC}$;*
2. *When $b_1 < b < b_2$, we have $\Delta\pi_A > \Delta\Pi_{SC}$ when $s < \frac{1+b}{8b}\left(8 - b - \sqrt{b^2+16}\right)$ and $\Delta\pi_A < \Delta\Pi_{SC}$ when $s > \frac{1+b}{8b}\left(8 - b - \sqrt{b^2+16}\right)$;*

*3. When $b > b_2$, we have $\Delta\pi_A < \Delta\Pi_{SC}$.*

*However, when $s \in \left(\frac{1+b}{1+2b}, 1\right)$, we always have $\Delta\pi_A < \Delta\Pi_{SC}$.*

The four regions as specified in $(\gamma, f)$ space with a given value of $b$ when $s \in \left(0, \frac{1+b}{1+2b}\right)$ are depicted in Figure **??**. It is interesting to see that the supply chain always benefits from retailer A selling the retailer brand through retailer B when the quality level of the retailer brand is very high. From the perspective of the supply chain as a whole, it is socially optimal to use the retailer brand to create competition in both markets, which reduces the margin charged by the supplier, therefore reduces double-marginalization. However, when the quality level is low, the story is not as simple as the former case.

## 5.4 Effect of Competition

We now extend our basic model analyzed in Section 5.3 by incorporating competition to further demonstrate the robustness of our key findings. The analysis in this section shows that retailer A's decision on selling the retailer brand through retailer B is also affected by the degree of competition between the two retailers. For expositional simplicity, we consider the case in which both retailers have no exclusive markets. That is, we consider the full competition case. In this case, we normalize the market size of market C to be one.

In sub-game $e = 0$, retailer B sets $q^C_{n,B}$ to maximize the profit function

$\pi_B^C$ as follows

$$\max_{q_{n,B}^C} \pi_B^C = \left[p_n\left(q_{p,A}^C, q_{n,A}^C + q_{n,B}^C; 1\right) - w_n\right] q_{n,B}^C. \tag{5.11}$$

Similarly, retailer A sets $q_{n,A}^C$ and $q_{p,A}^C$ to maximize the profit function $\pi_A^C$:

$$\max_{q_{n,A}^C,\, q_{p,A}^C} \pi_A^C = \left[p_n\left(q_{p,A}^C, q_{p,A}^C + q_{n,B}^C; 1\right) - w_n\right] q_{n,A}^C + p_p\left(q_{p,A}^C, q_{p,A}^C + q_{n,B}^C; 1\right) q_{p,A}^C. \tag{5.12}$$

We find the best responses of retailer A and retailer B in market C as follows:

$$\begin{cases} q_{n,A}^{C,1} = \frac{1}{6}\left(2 - \frac{(2+\gamma)w_n}{1-\gamma}\right), & q_{p,A}^{C,1} = \frac{w_H}{2-2\gamma}, \quad q_{n,B}^{C,1} = \frac{1-w_H}{3}, \quad \text{if } 0 \leq w_n \leq \frac{2-2\gamma}{2+\gamma}; \\ q_{n,A}^{C,2} = 0, & q_{p,A}^{C,2} = \frac{1+cw}{4-\gamma}, \quad q_{n,B}^{C,2} = \frac{2-\gamma-2\,w_n}{4-\gamma}, \quad \text{if } \frac{2-2\gamma}{2+\gamma} \leq w_n \leq \frac{2-\gamma}{2}; \\ q_{n,A}^{C,3} = 0, & q_{p,A}^{C,3} = \frac{1}{2}, \quad q_{n,B}^{C,3} = 0, \quad \text{if } \frac{2-\gamma}{2} \leq w_n \leq 1. \end{cases} \tag{5.13}$$

Notice that the region for retailer A selling the national brand is a subset of the region for retailer B selling it. Since retailer B has no other options, she will sell the national brand as long as the profit margin is not zero. However, for retaler A, she will sell the national brand only when the profit margin exceeds that of her own retailer brand.

Anticipating both retailers' optimal response on sales quantities for the national brand and retailer brand respectively, the supplier sets the wholesale price $w_n$ for the national brand to maximize his profit:

$$\max_{w_n} \Pi^C = w_n\left(q_{n,A}^C + q_{n,B}^C\right).$$

Substituting the retailers' best responses in equation (5.13) into supplier's profit function, we identify the supplier's optimal wholesale price for the national brand:

$$w_{n,C}^{0} = \begin{cases} \frac{2-2\gamma}{4-\gamma}, & \text{if } 0 < \gamma \leq \frac{2}{3}; \\ \frac{2-\gamma}{4} & \text{if } \frac{2}{3} \leq \gamma < 1. \end{cases}$$

The equilibrium quantities and profits are provided in Table 5.4.

In sub-game $e = 1$, retailer A sets $q_{n,A}^{C}$ and $q_{p,A}^{C}$ to maximize her profit $\pi_{A}^{C}$:

$$\begin{aligned} \pi_{A}^{C} &= \left[p_n\left(q_{p,A}^{C} + q_{p,B}^{C}, q_{n,A}^{C} + q_{n,B}^{C}; 1\right) - w_n\right] q_{n,A}^{C} + \\ &\quad p_p\left(q_{p,A}^{C} + q_{p,B}^{C}, q_{n,A}^{C} + q_{n,B}^{C}; 1\right) q_{p,A}^{C} + w_p q_{p,B}^{C}. \end{aligned} \tag{5.14}$$

Similarly, retailer B determines $q_{n,B}^{C}$ and $q_{p,B}^{C}$ to maximize her profit $\pi_{B}^{C}$:

$$\begin{aligned} \pi_{B}^{C} &= \left[p_n\left(q_{p,A}^{C} + q_{p,B}^{C}, q_{n,A}^{C} + q_{n,B}^{C}; 1\right) - w_n\right] q_{n,B}^{C} + \\ &\quad \left[p_p\left(q_{p,A}^{C} + q_{p,B}^{C}, q_{n,A}^{C} + q_{n,B}^{C}; 1\right) - w_p\right] q_{p,A}^{C}. \end{aligned} \tag{5.15}$$

Depend on the values of $w_n$ and $w_p$, the quantity decisions can be characterized in Table 5.5. The regions are depicted in Figure 5.2.

In stage three, retailer A sets the wholesale price $w_p$ for her retailer brand to maximize her profit for any given value of $w_n$, anticipating the outcome of the quantity competition in stage two. The detailed analysis can be found in the appendix. We present the equilibrium for this sub-game in the following lemma.

**Lemma 5.4.1.** *When retailer A and retailer B competing in the same market, there exist two threshold values $\gamma_1^C$ and $\gamma_2^C$, such that the equilibrium of the sub-game $e = 1$ can be characterized as follows:*

1. *When* $0 < \gamma \leq \gamma_1^C$, *the supplier sets* $w_n = \frac{2-2\gamma}{4-\gamma}$; *retailer A sets* $w_p = \frac{\gamma(1-\gamma)}{4-\gamma}$; *retailer A sells both products and retailer B sells only the national brand.*

2. *When* $\gamma_1^C < \gamma \leq \gamma_2^C$, *the supplier sets* $w_n = \frac{(8-\gamma)(1-\gamma)}{8+\gamma}$; *retailer A sets* $w_p = \frac{4\gamma(1-\gamma)}{8+\gamma}$; *retailer A sells only the retailer brand and retailer B sells only the national brand.*

3. *When* $\gamma_2^C < \gamma < 1$, *the supplier sets* $w_n = \frac{(20-\gamma)(1-\gamma)}{40-22\gamma}$; *retailer A sets* $w_p = \frac{\gamma(1-\gamma)(220-31\gamma)}{4(10-\gamma)(20-11\gamma)}$; *retailer A sells only the retailer brand and retailer B sells both products.*

Notice that we have $\gamma_1^C > \frac{2}{3}$. It means that retailer A will keep selling the national brand for higher quality level of the retailer brand. This is due to the fact that competition between the two retailers provides leverage for the national brand supplier. It is noteworthy to point out that the strategic effect of selling the retailer brand through retailer B still exists even under full competition. That is, selling the retailer brand induces a lower wholesale price for the national brand. This is clear from the comparison of the equilibrium wholesale prices in the two sub-games.

| | Outcome of Sub-game $e=0$ | | Comparative Statics w.r.t. $(\gamma,\, b)$ | |
|---|---|---|---|---|
| | $0<\gamma<\hat{\gamma}_0$ | $\hat{\gamma}_0 \leq \gamma < 1$ | $0<\gamma<\hat{\gamma}_0$ | $\hat{\gamma}_0 \leq \gamma < 1$ |
| $w_n$ | $\frac{(1+b)(1-\gamma)}{2+2b(1-\gamma)}$ | $\frac{1}{2}$ | $(-,+)$ | $(0,0)$ |
| $\Pi$ | $\frac{(1+b)^2(1-\gamma)}{8+8b(1-\gamma)}$ | $\frac{b}{8}$ | $(-,+)$ | $(0,+)$ |
| $\pi_A$ | $\frac{1+3\gamma+b(1-\gamma)(2+b+4\gamma)}{16(1+b-b\gamma)^2}$ | $\frac{\gamma}{4}$ | $(+,-)$ | $(+,0)$ |
| $\pi_B$ | $\frac{b(1+b+\gamma-b\gamma)^2}{16(1+b-b\gamma)^2}$ | $\frac{b}{16}$ | $(+,+)$ | $(0,+)$ |
| $q_{n,A}$ | $\frac{1+b-2b\gamma}{4(1+b-b\gamma)}$ | $0$ | $(-,-)$ | $(0,0)$ |
| $q_{p,A}$ | $\frac{1+b}{4(1+b-b\gamma)}$ | $\frac{1}{2}$ | $(+,+)$ | $(0,0)$ |
| $q_{n,B}$ | $\frac{b(1+b+\gamma-b\gamma)}{4(1+b-b\gamma)}$ | $\frac{b}{4}$ | $(+,+)$ | $(0,+)$ |

Table 5.2: Equilibrium and comparative statics for sub-game $e=0$.

| | Outcome of Sub-game $e = 1$ | Comparative Statics w.r.t. $(\gamma,\, b)$ |
|---|---|---|
| $w_n$ | $\frac{(1+b)(1-\gamma)}{2+b(2-\gamma)}$ | $(-, +)$ |
| $w_p$ | $\frac{(1+b)(1-\gamma)\gamma}{4+2b(2-\gamma)}$ | $(\pm, +)$ |
| $\Pi$ | $\frac{(1+b)^2(1-\gamma)}{8+4b(2-\gamma)}$ | $(-, +)$ |
| $\pi_A$ | $\frac{1}{4} - \frac{(1+b)(1-\gamma)[6(1+b)-b(5+b)\gamma]}{8(2+b(2-\gamma))^2}$ | $(\pm, \pm)$ |
| $\pi_B$ | $\frac{b\left[4(1+b)^2+(1+b)(9+b)q+(1-b)(3+b)q^2\right]}{16(2+b(2-\gamma))^2}$ | $(+, +)$ |
| $q_{n,A}$ | $\frac{1+b-b\gamma}{4+4b-2b\gamma}$ | $(-, -)$ |
| $q_{p,A}$ | $\frac{1+b}{4+4b-2b\gamma}$ | $(+, +)$ |
| $q_{n,B}$ | $\frac{b(2+\gamma+b(2-\gamma))}{8+4b(2-\gamma)}$ | $(+, +)$ |
| $q_{p,B}$ | $\frac{b(1+b)}{8+4b(2-\gamma)}$ | $(+, +)$ |

Table 5.3: Equilibrium and comparative statics for sub-game $e = 1$.

| | Outcome of Sub-game $e = 0$ | | Comparative Statics w.r.t. $\gamma$ | |
|---|---|---|---|---|
| | $0 < \gamma < \frac{2}{3}$ | $\frac{2}{3} \leq \gamma < 1$ | $0 < \gamma < \frac{2}{3}$ | $\frac{2}{3} \leq \gamma < 1$ |
| $w_n$ | $\frac{2-2\gamma}{4-\gamma}$ | $\frac{2-\gamma}{4}$ | - | - |
| $\Pi$ | $\frac{2-2\gamma}{3(4-\gamma)}$ | $\frac{(2-\gamma)^2}{8(4-\gamma)}$ | - | - |
| $\pi_A$ | $\frac{4+(13-8\gamma)\gamma}{9(4-\gamma)^2}$ | $\frac{\gamma(6-\gamma)^2}{16(4-\gamma)^2}$ | + | + |
| $\pi_B$ | $\frac{(2+\gamma)^2}{9(4-\gamma)^2}$ | $\frac{(2-\gamma)^2}{4(4-\gamma)^2}$ | + | - |
| $q_{n,A}$ | $\frac{2-2\gamma}{3(4-\gamma)}$ | 0 | - | - |
| $q_{p,A}$ | $\frac{1}{4-\gamma}$ | $\frac{6-\gamma}{4(4-\gamma)}$ | + | + |
| $q_{n,B}$ | $\frac{2+\gamma}{3(4-\gamma)}$ | $\frac{2-\gamma}{2(4-\gamma)}$ | + | - |

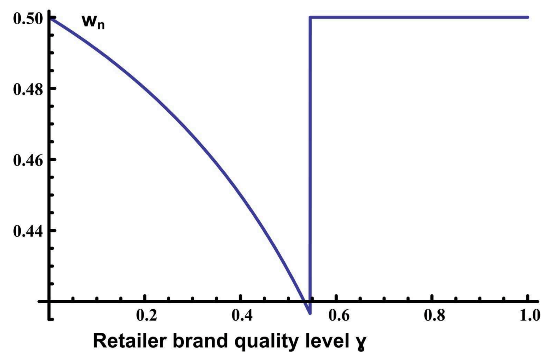
Table 5.4: The equilibrium of sub-game $e = 0$ in the full competition case.

| Regions | $q_{n,A}^C$ | $q_{p,A}^C$ | $q_{n,B}^C$ | $q_{p,B}^C$ |
|---|---|---|---|---|
| $R_{w,C}^1$ | $\frac{1}{3}\left(1-\frac{w_n+w_p}{1-\gamma}\right)$ | $\frac{\gamma w_n+w_p}{3\gamma(1-\gamma)}$ | $\frac{1}{3}\left(1-\frac{w_n-2w_p}{1-\gamma}\right)$ | $\frac{\gamma w_n-2w_p}{3\gamma(1-\gamma)}$ |
| $R_{w,C}^2$ | $\frac{1}{6}\left(2-\frac{(2+\gamma)w_n}{1-\gamma}\right)$ | $\frac{w_H}{2(1-\gamma)}$ | $\frac{1}{3}(1-w_n)$ | $0$ |
| $R_{w,C}^3$ | $0$ | $\frac{1}{3}\left(1+\frac{w_p}{\gamma}\right)$ | $\frac{1}{2}\left(1-\frac{w_n-w_p}{1-\gamma}\right)$ | $\frac{1}{6}\left(\frac{3\gamma w_n-(4-\gamma)w_p}{q(1-q)}-1\right)$ |
| $R_{w,C}^4$ | $0$ | $\frac{1+w_n}{4-\gamma}$ | $\frac{2-\gamma-2w_n}{4-\gamma}$ | $0$ |
| $R_{w,C}^5$ | $0$ | $\frac{1}{3}\left(1+\frac{w_p}{\gamma}\right)$ | $0$ | $\frac{1}{3}\left(1-\frac{2w_p}{\gamma}\right)$ |
| $R_{w,C}^6$ | $0$ | $\frac{1}{2}$ | $0$ | $0$ |

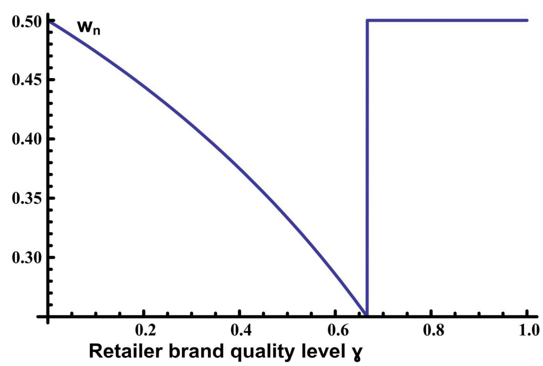
Table 5.5: Retailer A and B's quantity decisions in market C.

The six regions are defined as follows:

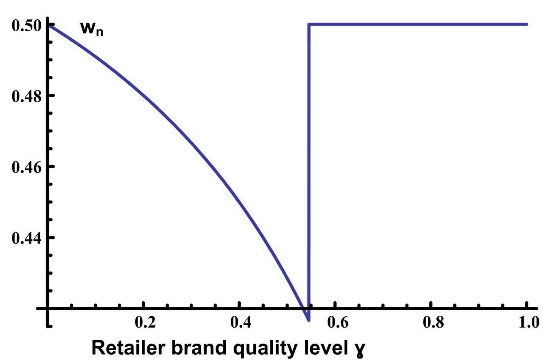
$$
\begin{aligned}
R_{w,C}^1 &= \left\{ \begin{array}{l} 0 \le w_n \le \frac{2(1-\gamma)}{2+\gamma} \textit{ and } 0 \le w_p \le \frac{\gamma w_n}{2}, \textit{ or} \\ \frac{2(1-\gamma)}{2+\gamma} \le w_n \le 1-\gamma \textit{ and } 0 \le w_p \le 1-\gamma-w_n \end{array} \right\}, \\
R_{w,C}^2 &= \left\{ 0 \le w_n \le \frac{2(1-\gamma)}{2+\gamma} \textit{ and } \frac{\gamma w_n}{2} \le w_p \le \gamma \right\}, \\
R_{w,C}^3 &= \left\{ \begin{array}{l} \frac{2(1-\gamma)}{2+\gamma} \le w_n \le 1-\gamma \textit{ and } 1-\gamma-w_n \le w_p \le \frac{\gamma(3w_n-1+\gamma)}{4-\gamma}, \textit{ or} \\ 1-\gamma \le w_n \le \frac{2-\gamma}{2} \textit{ and } w_n-1+\gamma \le w_p \le \frac{\gamma(3w_n-1+\gamma)}{4-\gamma} \end{array} \right\}, \\
R_{w,C}^4 &= \left\{ \frac{2(1-\gamma)}{2+\gamma} \le w_n \le \frac{2-\gamma}{2} \textit{ and } \frac{\gamma(3w_n-1+\gamma)}{4-\gamma} \le w_p \le \gamma \right\}, \\
R_{w,C}^5 &= \left\{ \begin{array}{l} 1-\gamma \le w_n \le \frac{2-\gamma}{2} \textit{ and } 0 \le w_p \le w_n-1+\gamma, \textit{ or} \\ \frac{2-\gamma}{2} \le w_n \le 1 \textit{ and } 0 \le w_p \le \frac{\gamma}{2} \end{array} \right\}, \\
R_{w,C}^6 &= \left\{ \frac{2-\gamma}{2} \le w_n \le 1 \textit{ and } \frac{\gamma}{2} \le w_p \le \gamma \right\}.
\end{aligned}
$$




(a) $b = 0.5$




(b) $b = 1$




(c) $b = 5$

Figure 5.1: Equilibrium wholesale price $w_n$ for the national brand in sub-game $e = 0$.

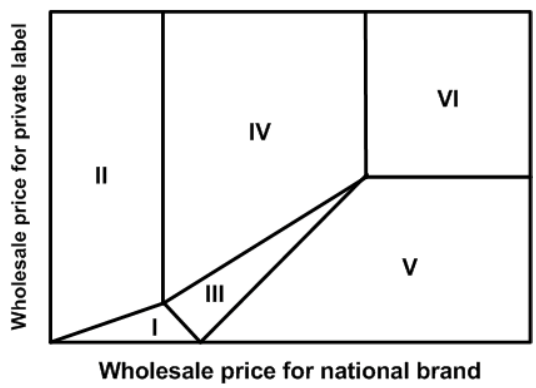



Figure 5.2: Retailer A and B's quantity decisions in market C for any given wholesale prices for the national brand and the retailer brand products.

# Chapter 6

# Conclusions and Directions for Future Research

In this dissertation, we study varies operational and marketing issues in supply chain management in the presence of vertically differentiated products, especially in the context of interactions between private labels and national brands. We present game theoretical models to study how the presence of vertically differentiated products may change the interactions among supply chain members and what are the implications to supply chain efficiency. We study the problem of private label development and distribution. We also investigate the strategic interactions between product line efficiency and competition when there are strategic suppliers. The analysis of these models show that it is important for firms to understand the strategic implications of competition and product line efficiency. Competition and product line inefficiency cannot be simply viewed as "bad" things and spend valuable resources to defend them. It is critical for the firms' success to better understand the strategic interactions among supply chain members when competition and product line inefficiency emerge in firms' operations practice, especially when vertically differentiated products are at presence.

In Chapter 3, we recognize the role that is played by a retailer's ability to develop and produce her own private label product in coordinating a decentralized supply chain. In contrast to the existing literature, we explicitly consider the fact that development costs are often a prerequisite for a retailer selling her own private label, and we also recognize that a retailer's marginal costs may differ from those of a national brand manufacturer, both absolutely and relative to the qualities of their products. We establish that, in a decentralized supply chain, the retailer will develop structurally efficient private labels (if she has the opportunity to do so), but she will also develop structurally inefficient private labels. While the development of these structurally inefficient private labels always reduces the profit of the national brand manufacturer, they may or may not lead to higher overall supply chain profits. Chapter 4 shows that competition from a lower quality entrant may benefit an incumbent by inducing a lower wholesale price of the critical component adopted by both the incumbent and the entrant. We are also able to show that limited capacity on the incumbent's high end product not only creates inefficiency in product line, but also can be a credible signal to the critical component suppliers to induce a lower wholesale price, which implies that product line inefficiency may play a similar strategic role as a lower quality entrant without demand cannibalization. Finally, in Chapter 5, we investigate a retailer's decision of whether to keep their retailer brand private or sell it through competitors. Our analysis recognize that the perceived quality level of the retailer brand, the relative market size of the competitor, and the degree of competition play important role in making the decision. In summary, the theory and insights developed in this dissertation are applicable to operations and marketing managers' strategic decision making in various contexts.

Our work is by no means an exhaustive study of the issues we consider. There are several ways in which one could extend and enrich the models and analysis in this research. Inclusion of empirical testing of our analytical results, a richer model of contracts among supply chain members and adoption of more general models of competition could provide a more nuanced understanding of the strategic impact of the presence of vertically differentiated products on a supply chain. Future research could endogenize the product quality decision and examine the equilibrium product characteristics. In addition, some national brand manufacturers have their own direct selling channels. The interactions between the direct channel and traditional channels with private labels is also an avenue for future research. In conclusion, while this dissertation makes some important progress in understanding the interactions among supply chain members in the presence of vertically differentiated products, it is by no means a complete study of this field. Hopefully, this initial work can lead to many interesting future research streams.

# Appendices

# Appendix A

# Proofs for Chapter 2

**Proof for Lemma 2.2.1:**

*Proof.* When $\theta_1 \leq \theta_2$, we have

$$\begin{aligned}
\theta_{1,2} - \theta_2 &= \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_2}{q_2} \\
&= \frac{q_1}{q_2 - q_1}\left(\frac{p_2}{q_2} - \frac{p_1}{q_1}\right) \\
&\geq 0.
\end{aligned}$$

Thus we have proved result 1. Similarly we can show that result 2 is also true. □

**Proof of Lemma 2.2.2:**

*Proof.* If $(p_1, p_2) \in R_1$, we have $\theta_1 \geq \theta_2 \geq \theta_{1,2}$. Thus, only consumers with valuation $\theta \geq \theta_1$ purchases product 1. There are not consumers buying product 2. Thus, we have proved result 1. Results 2 and 3 can be shown similarly. □

# Appendix B

# Proofs for Chapter 3

**Proof of Theorem 3.2.1:**

*Proof.* In sub-game $d = 0$, the vertically integrated channel chooses retail price $p_n$ for the national brand to maximize the profit $\Pi_{SC}(p_n) = \pi(C, p_n, q)$ and the solution is given by $p_n = \frac{1+KC}{1+K}$. Denote the corresponding profit by $\Pi^0_{SC}$.

In sub-game $d = 1$, the vertically integrated channel chooses retail prices $p_n$ and $p_p$ for the national brand and the private label to maximize the total profit $\Pi_{SC}(p_n, p_p) = \pi(C, p_n, p_p)$. The profit is jointly concave in $p_n$ and $p_p$ in each of the three regions $R_N$, $R_B$, and $R_P$. By taking the first order derivatives with respect to $p_n$ and $p_p$, we have the following solution: $p_n = \frac{1+KC}{1+K}$ and $p_p = \frac{q+Kc}{1+K}$. When $RPM \leq q$, we have $(p_n, p_p) \in R_N$ and the solution is the same as in sub-game $d = 0$; when $q < RPM < 1$, we have $(p_n, p_p) \in R_B$; and when $RPM \geq 1$, we have $(p_n, p_p) \in R_P$. Denote the corresponding profit by $\Pi^1_{SC}$.

Obviously we have $\Pi^1_{SC} = \Pi^0_{SC}$ when $RPM \leq q$ and $\Pi^1_{SC} > \Pi^0_{SC}$ when $RPM > q$. The vertically integrated channel develops the private label if and only if $\Pi^1_{SC} - \Pi^0_{SC} > g$. Thus, when $RPM \leq q$, the channel never develops the private label ($d^{FB} = 0$) and sells only the national brand ($p^{FB}_n = \frac{1+KC}{1+K}$).

When $q < RPM < 1$, let $\hat{g}_b = \Pi_{SC}^1 - \Pi_{SC}^0$. When $g < \hat{g}_b$, the channel develops the private label ($d^{FB} = 1$) and sells both products ($p_n^{FB} = \frac{1+KC}{1+K}$ and $p_p^{FB} = \frac{q+Kc}{1+K}$); otherwise, the private label is not developed ($d^{FB} = 0$) and only the national brand is sold ($p_n^{FB} = \frac{1+KC}{1+K}$). Similarly, when $RPM \geq 1$, define $\hat{g}_p = \Pi_{SC}^1 - \Pi_{SC}^0$. When $g < \hat{g}_p$, the channel develops the private label ($d^{FB} = 1$) and sells only it ($p_p^{FB} = \frac{q+Kc}{1+K}$); otherwise, the private label is not developed ($d^{FB} = 0$) and only the national brand is sold ($p_n^{FB} = \frac{1+KC}{1+K}$).

From the definition, we have

$$\frac{\partial \hat{g}_b}{\partial c} = \frac{\partial \left(\Pi_{SC}^1 - \Pi_{SC}^0\right)}{\partial c} = \left[\frac{K\left(1-C-q+c\right)}{\left(1+K\right)\left(1-q\right)}\right]^K - \left[\frac{K\left(q-c\right)}{\left(1+K\right)q}\right]^K < 0$$

and

$$\frac{\partial \hat{g}_b}{\partial C} = \frac{\partial \left(\Pi_{SC}^1 - \Pi_{SC}^0\right)}{\partial C} = \left[\frac{K\left(1-C\right)}{1+K}\right]^K - \left[\frac{K\left(1-C-q+c\right)}{\left(1+K\right)\left(1-q\right)}\right]^K > 0$$

since $q < RPM < 1$. Thus, $\hat{g}_b$ is decreasing in $c$ and increasing in $C$ when $q < RPM < 1$. Similarly we can show that $\hat{g}_p$ is decreasing in $c$ and increasing in $C$ when $RPM \geq 1$. $\square$

**Proof of Theorem 3.3.1:**

*Proof.* The results regarding the national brand production cost $C$ are obvious. We establish the results regarding the parameter $K$. For the retailer's profit $\pi^N$, by taking derivative with respect to $K$, we have

$$\frac{\partial \pi^N}{\partial K} = \frac{1+K}{K^2}\left[\frac{K^2\left(1-C\right)}{\left(1+K\right)^2}\right]^{1+K}\left(\frac{1}{1+K} + \log\frac{K^2\left(1-C\right)}{\left(1+K\right)^2}\right).$$

It is sufficient to show that the term in the parenthesis is negative for any values of $K > 0$. In fact, when $K \to +\infty$, the term goes to $\log(1-C)$ which is negative since $0 < C < 1$. Furthermore, the derivative of the term with respect to $K$ is

$$\frac{\partial}{\partial K}\left(\frac{1}{1+K} + \log\frac{K^2(1-C)}{(1+K)^2}\right) = \frac{2+K}{K(1+K)^2} > 0.$$

Thus, the term is strictly increasing and negative and the retailer's profit $\pi^N$ is decreasing in $K$. We can prove that $\Pi^N$, $\Pi^N_{SC}$, and $Q^N_n$ are also decreasing in $K$ in a similar way. □

**Proof of Lemma 3.3.2:**

*Proof.* Denote $p_n(w) = \frac{1+Kw}{1+K}$, $p_p(w) = \frac{q+Kc}{1+K}$, $\pi^n(w) = \pi(w, p_n(w), q)$, and $\pi^b(w) = \pi(w, p_n(w), p_p(w))$.

When $0 \leq w \leq \frac{c}{q}$, we have $(p_n(w), p_p(w)) \in R_N$. From the definition of $\pi(w, p_n, p_p)$, we have $\pi(w, p_n, p_p) = \pi(w, p_n, q)$ if $(p_n, p_p) \in R_N$. The maximizer of $\pi(w, p_n, q)$ is given by $p_n = p_n(w)$. Thus, $(p_n(w), q)$ will be a maximizer of $\pi(w, p_n, p_p)$ when $(p_n, p_p) \in R_N$. Next, we show that $\pi^n(w) \geq \pi(w, p_n, p_p)$ if $(p_n, p_p) \in R_B$. Given $p_p$, the unique solution for $\partial\pi(w, p_n, p_p)/\partial p_n = 0$ is given by

$$p_n(p_p|w) = \frac{1 + p_p - q + K(p_p - c + w)}{1+K}.$$

We have $\partial\pi(w, p_n, p_p)/\partial p_n > 0$ when $p_n < p_n(p_p|w)$ and $\partial\pi(w, p_n, p_p)/\partial p_n < 0$ when $p_n > p_n(p_p|w)$ for $(p_n, p_p) \in R_B$. Thus, $\pi(w, p_n, p_p)$ is unimodal for any given value of $p_p$ when $(p_n, p_p) \in R_B$. In addition, the unique solution for $\partial\pi(w, p_n, p_p)/\partial p_p = 0$ on the line $p_n = p_n(p_p|w)$ is given by $p_p = \frac{q+Kc}{1+K}$, which gives $p_n = \frac{1+Kw}{1+K}$. Thus, given $p_n = p_n(p_p|w)$, the function $\pi(w, p_n, p_p)$ is unimodal in $p_p$. Therefore, if $(p_n, p_p) \in R_B$, we have

$$\pi(w, p_n, p_p) \leq \pi(w, p_p/q, p_p) \leq \pi(w, p_n(w), qp_n(w)) = \pi^n(w)$$

if $p_p/q \geq p_n(p_p|w)$. The first inequality is due to the fact that $\pi(w, p_n, p_p)$ is decreasing in $p_n$ when $p_n \leq p_n(p_p|w)$. The second inequality is due to the fact that $\pi(w, p_n, qp_n) = \pi(w, p_n, p_p)$ is maximized at $p_n = p_n(w)$ when $(p_n, p_p) \in R_N$. If $p_p/q \leq p_n(p_p|w)$, then we have

$$\begin{aligned}
\pi(w, p_n, p_p) &\leq \pi(w, p_n(p_p|w), p_p) \\
&\leq \pi\left(w, p_n\left(\frac{q(1-Kc-q-Kw)}{1+K}\right), \frac{q(1-Kc-q-Kw)}{1+K}\right) \\
&= \pi\left(w, \frac{1-Kc-q-Kw}{1+K}, \frac{q(1-Kc-q-Kw)}{1+K}\right) \leq \pi^n(w).
\end{aligned}$$

The first inequality is due to the fact that $\pi(w, p_n, p_p)$ is increasing in $p_n$ if $p_n \leq p_n(p_p|w)$. The second inequality is due to the fact that $p_p \leq \frac{q(1-Kc-q-Kw)}{1+K}$ if $p_n(p_p|w) \geq p_p/q$. Thus, we have proved that $\pi^n(w) \geq \pi(w, p_n, p_p)$ if $(p_n, p_p) \in R_B$. When $(p_n, p_p) \in R_P$, we have $\pi(w, p_n, p_p) = \pi(w, p_p + 1 - q, p_p)$ and $(p_p + 1 - q, p_p) \in R_B$. Thus, we have $\pi^n(w) \geq \pi(w, p_p + 1 - q, p_p) = \pi(w, p_n, p_p)$ when $(p_n, p_p) \in R_P$. Hence, we have proved that $(p_n(w), q)$ is a global maximizer of $\pi(w, p_n p_p)$ when $0 \leq w \leq \frac{c}{q}$.

When $\frac{c}{q} < w < 1 + c - q$, we have $(p_n(w), p_p(w)) \in R_B$ which is the first order solution of $\max \pi(w, p_n, p_p)$ when $(p_n, p_p) \in R_B^c$, where $R_B^c$ is the set $R_B$ union of the two boundaries $qp_n = p_p$ and $p_n = p_p - 1 + q$. The second order conditions can be checked as follows:

$$\begin{aligned}
\frac{\partial^2 \pi^b (w, p_n, p_p)}{\partial p_n^2} \Big|_{(p_n(w), p_p(w))} &= -(1+K) \left[ \frac{K(1+c-q-w)}{(1+K)(1-q)} \right]^{K-1} < 0. \\
\left| H\pi^b (w, p_n, p_p) \right| \Big|_{(p_n(w), p_p(w))} &= (1+K)^2 \left[ \frac{K(q-c)}{(1+K)q} \right]^{K-1} \left[ \frac{K(1+c-q-w)}{(1+K)(1-q)} \right]^{K-1} > 0.
\end{aligned}$$

Thus, $\pi^b(w) \geq \pi(w, p_n, p_p)$ when $(p_n, p_p) \in R_B^c$. If $(p_n, p_p) \in R_N$, we have $\pi^b(w) \geq \pi(w, p_n, p_p)$ since $\pi(w, p_n, p_p) = \pi(w, p_n, qp_n)$ and $(p_n, qp_n) \in R_N^c$. Similarly, if $(p_n, p_p) \in R_P$, we have

$$\pi^b(w) - \pi(w, p_n, p_p) = \pi^*(w) - \pi(w, p_p + 1 - q, p_p) \geq 0$$

since $(p_p + 1 - q, p_p) \in R_B^c$. Therefore, we have proved that $(p_n(w), p_p(w))$ is a global maximizer of $\pi(w, p_n p_p)$ when $(p_n(w), p_p(w)) \in R_B$.

Similarly, we can prove the results for $w \geq 1 + c - q$. ☐

**Proof of Theorem 3.3.3:**

*Proof.* From Lemma 3.3.2, the manufacturer's profit can be written as

$$\Pi(w) = \begin{cases} Q_n(p_n^1(w), q)(w - C), & if\ 0 \leq w \leq \frac{c}{q}; \\ Q_n\left(p_n^1(w), p_p^1(w)\right)(w - C), & if\ \frac{c}{q} \leq w \leq 1 + c - q; \\ 0, & if\ 1 + c - q \leq w \leq 1. \end{cases}$$

We identify the optimal wholesale price in three cases respectively: a) $0 \leq C \leq \frac{c}{q}$; b) $\frac{c}{q} \leq C \leq 1 + c - q$; and c) $1 + c - q \leq C \leq 1$.

We first consider case (c). The condition of case c is equivalent to $RPM \in P$. In this case, the manufacturer's profit is zero. Thus, there is no participation from the manufacturer and the retailer sells the private label only. In case (b), the condition is equivalent to $q \leq RPM \leq 1$ which is a subset of $NPH$. The unique solution for the first order condition of the second branch is $w = w^b = \frac{1+KC-q+c}{1+K}$. Since $1 - C + c - q \geq 0$, the manufacturer's profit $\Pi(w)$ is concave in $w$. Thus, the optimal wholesale price is $w = w^b$ and the retailer sells both the national brand and the private label. In case (a), the unique solution for the first order condition of the first branch is $w = w^n = \frac{1+KC}{1+K} > w^b$. The profit $\Pi(w)$ is concave in each range of the values for $w$. Thus, we have the following cases: i) If $\frac{c}{q} \leq w^b$, then $\Pi(w)$ is increasing in $w$ when $0 \leq w \leq \frac{c}{q}$. In addition, we have $\frac{c}{q} \leq w^b \leq 1 + c - q$. Thus, the optimal wholesale price is $w = w^b$ in this case. The condition $\frac{c}{q} \leq w^b \leq 1 + c - q$ is equivalent to $\frac{Kq}{1+K-q} \leq RPM \leq q$; ii) If $w^b \leq \frac{c}{q} \leq w^n$, then $\Pi(w)$ is increasing in $w$ when $0 \leq w \leq \frac{c}{q}$ and decreasing in $w$ when $\frac{c}{q} \leq w \leq 1 + c - q$. Thus, the optimal wholesale price is $w = \frac{c}{q}$. The condition $w^b \leq \frac{c}{q} \leq w^n$ is equivalent to $RPM \in NPM$; iii) If $w^n \leq \frac{c}{q}$, then $\Pi(w)$ is decreasing in $w$ when $\frac{c}{q} \leq w \leq 1 + c - q$. Thus, the optimal value for $w$ is $w = w^n$. The condition is equivalent to $RPM \in N$. Combining the results with the retailer's best response in Lemma 3.3.2, we have proved the theorem. □

**Proof of Corollary 3.3.4:**

*Proof.* The results are obvious from the expression of each threshold values. □

**Proof of Corollary 3.3.5:**

*Proof.* The results can be obtained by taking derivatives with respect to the corresponding parameters. $\square$

**Proof of Theorem 3.3.6:**

*Proof.* Define $c_L^B = \max\{0,\ q + C - 1\}$, $c_M^B = \max\left\{0,\ \frac{q(1-q+KC)}{1-q+K}\right\}$, and $c_U^B = \frac{q(1+KC)}{1+K}$. When $RPM \in P$, we have $0 \le c \le c_L^B$; when $RPM \in NPH$, we have $c_L^B < c < c_M^B$; when $RPM \in NPM$, we have $c_M^B \le c \le c_U^B$; and when $RPM \in N$, we have $c_U^B < c \le q$. We have $w^N = \frac{1+KC}{1+K}$ and

$$w^B = \begin{cases} \frac{1+KC-q+c}{1+K}, & c_L^B < c < c_M^B; \\ \frac{c}{q}, & c_M^B \le c \le c_U^B; \\ \frac{1+KC}{1+K}, & c_U^B < c \le q. \end{cases}$$

When $0 \le c \le c_L^B$, the retailer does not carry the national brand. The first result follows.

For the second result, we have $Q_n^N = \left[\frac{K^2(1-C)}{(1+K)^2}\right]^K$ and

$$Q_n^B = \begin{cases} \left[\frac{K^2(1-C-q+c)}{(1+K)^2(1-q)}\right]^K, & c_L^B < c < c_M^B; \\ \left[\frac{K(q-c)}{(1+K)q}\right]^K, & c_M^B \le c \le c_U^B; \\ \left[\frac{K^2(1-C)}{(1+K)^2}\right]^K, & c_U^B < c \le q. \end{cases}$$

When $0 \le c \le c_L^B$, the retailer does not carry the national brand. It is easy to check that $Q_n^B$ is a continuous function of $c$ on $\left[c_L^B, q\right]$. When $c = c_L^B$, we have $Q_n^B < Q_n^N$. When $c = c_M^B$, we have $Q_n^B > Q_n^N$. And when $c \in \left(c_U^B, q\right]$, we have $Q_n^B = Q_n^N$. It is obvious that $Q_n^B$ is increasing in $\left(c_L^B, c_M^B\right)$ and decreasing in $\left[c_M^B, c_U^B\right]$. Thus, there exists a unique value of $c$ in $\left[c_L^B, c_M^B\right]$, denote by $c_Q$, such that $Q_n^B = Q_n^N$ when $c = c_Q$, $Q_n^B < Q_n^N$ when $c < c_Q$, and $Q_n^B \geq Q_n^N$ when $c \geq c_Q$.

The derivative of $\Pi^N - \Pi^B$ with respect to $c$ is always nonpositive. When $c = q$, we have $\Pi^N - \Pi^B = 0$. Thus, we always have $\Pi^B \leq \Pi^N$.

Denote $C_L^B = \max\left\{0, \frac{c}{q} - \frac{q-c}{qK}\right\}$, $C_M^B = \max\left\{0, \frac{c}{q} - \frac{(q-c)(1-q)}{qK}\right\}$, and $C_U^B = 1 + c - q$. For the result regarding the supply chain's profit, we only present the proof for the case in which $C_L^B \geq 0$. The proof for other cases is similar. We prove the results in each interval of the values for $C$. When $C \in \left[0, C_L^B\right]$, we have $\Pi_{SC}^B - \Pi_{SC}^N = 0$.

When $C \in \left[C_L^B, C_M^B\right]$, we show that $\Pi_{SC}^B - \Pi_{SC}^N$ is equal to zero at $C = C_L^B$, positive at $C = C_M^B$, and concave in $C$. Thus we can conclude that $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C \in \left(C_L^B, C_M^B\right]$. In fact, we have $\Pi_{SC}^B - \Pi_{SC}^N = 0$ at $C = C_L^B$. When $C = C_M^B$, we have

$$
\begin{aligned}
\frac{\Pi_{SC}^B}{\Pi_{SC}^N}\bigg|_{C=C_M^B} &= \frac{1 - q + K(2-q)}{1+2K}\left[\frac{1+K}{1+K-q}\right]^{1+K} \\
&= \frac{1 - q + K(2-q)}{1+2K}\left[1 + \frac{q}{1+K-q}\right]^{1+K} \\
&\geq \frac{1 - q + K(2-q)}{1+2K}\left[1 + \frac{q(1+K)}{1+K-q}\right] \\
&> \frac{1 - q + K(2-q)}{1+2K}\left[1 + \frac{q(1+K)}{1+K-q+K(1-q)}\right] = 1.
\end{aligned}
$$

Taking the second order derivative of $\Pi_{SC}^B - \Pi_{SC}^N$ with respect to $C$, we have

$$
\frac{\partial^2\left(\Pi_{SC}^B - \Pi_{SC}^N\right)}{\partial C^2} = -\frac{K(1+2K)}{(1-C)(1+K)}\left[\frac{K^2(1-C)}{(1+K)^2}\right]^K < 0.
$$

Thus, $\Pi_{SC}^B - \Pi_{SC}^N$ is concave in $C$. Hence, we have $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C \in \left(C_L^B, C_M^B\right]$.

When $C \in \left[C_M^B, C_U^B\right]$, we show that $\Pi_{SC}^B - \Pi_{SC}^N$ is unimodal in $C$, the minimizer is $C = \frac{c}{q}$, and $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C = \frac{c}{q}$. Thus we can conclude that $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C \in \left[C_M^B, C_U^B\right]$ since $\frac{c}{q} \in \left[C_M^B, C_U^B\right]$. In fact, the unique solution for the first order condition is $C = \frac{c}{q}$. By taking the second order derivative of $\Pi_{SC}^B - \Pi_{SC}^N$ with respect to $C$ and evaluate it at $C = \frac{c}{q}$, we have

$$\frac{\partial^2 \left(\Pi_{SC}^B - \Pi_{SC}^N\right)}{\partial C^2} \Big|_{C=\frac{c}{q}} = \frac{K^3 (1+2K)\, q}{(1+K)^3 (1-q)} \left[\frac{K^2 (q-c)}{(1+K)^2 q}\right]^{K-1} > 0.$$

Hence, the derivative of $\Pi_{SC}^B - \Pi_{SC}^N$ with respect to $C$ is negative when $C < \frac{c}{q}$ and positive when $C > \frac{c}{q}$, which implies that $\Pi_{SC}^B - \Pi_{SC}^N$ is unimodal in $C$ when $C \in \left[C_M^B, C_U^B\right]$. Now, it suffices to show that $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C = \frac{c}{q}$. In fact, we have

$$\begin{aligned} \frac{\Pi_{SC}^B}{\Pi_{SC}^N} \Big|_{C=\frac{c}{q}} &= 1 + q\left(\frac{1+K}{1+2K}\left[1+\frac{1}{K}\right]^K - 1\right) \\ &\geq 1 + q\left(\frac{1+K}{1+2K}\left[1+\frac{K}{K}\right] - 1\right) > 1. \end{aligned}$$

Thus, we have $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C = \frac{c}{q}$.

When $C \in \left[C_U^B, 1\right]$, the supply chain profit $\Pi_{SC}^B$ is constant in $C$ and $\Pi_{SC}^N$ is decreasing in $C$. Thus, it is sufficient to show that $\Pi_{SC}^B - \Pi_{SC}^N > 0$ when $C = C_U^B$:

$$\frac{\Pi_{SC}^B}{\Pi_{SC}^N} \Big|_{C=C_U^B} = \frac{1+K}{1+2K}\left[\frac{1+K}{Kq}\right]^K > \frac{1+K}{1+2K}\left[\frac{1+K}{K}\right]^K > 1.$$

Therefore, we have proved that $\Pi_{SC}^{B} \geq \Pi_{SC}^{N}$ and $\Pi_{SC}^{B} > \Pi_{SC}^{N}$ when $C > C_L^B$.

Combining the above results regarding the manufacturer and the supply chain's profits, we have

$$\pi^B - \pi^N = \Pi_{SC}^{B} - \Pi^B - \Pi_{SC}^{N} + \Pi^N \geq \Pi_{SC}^{B} - \Pi_{SC}^{N}.$$

□

**Proof of Theorem 3.3.7:**

*Proof.* The retailer's equilibrium private label development decision is obtained by comparing the profits in sub-game $d = 0$ and $d = 1$ taking into account the fixed development cost $g$. Thus, if and only if $\pi^B - \pi^N > g$, the retailer develops the private label. When $RPM \in N$, the private label has no effect on retailer's profit. Thus, the retailer does not develop it regardless $g$. When $RPM \in NPM$, let $\bar{g}_n = \pi^B - \pi^N$ at $w^* = \frac{c}{q}$ and $p_n^* = \frac{q+Kc}{q+Kq}$. If $g < \bar{g}_n$, the increase in profit with private label is more than offsetting the fixed cost $g$. Thus the retailer develops the private label. Otherwise, the retailer does not. Similarly, we can derive the rest of the results by combining the results in section 3.3.1 and Theorem 3.3.3 and 3.3.6. □

**Proof of Theorem 3.3.8:**

*Proof.* From Theorem 3.3.6, we have $\Delta^R \geq \Delta^{SC}$. Thus, it is sufficient to prove that $\Delta^{SC} \geq \Delta^{VI}$. In fact, we have $\Delta^{VI} = 0$ and $\Delta^{SC} \geq 0$ when $0 \leq C \leq \frac{c}{q}$.

When $\frac{c}{q} \leq C \leq 1 + c - q$, we have

$$\Delta^{SC} - \Delta^{VI} = (1 + K) \left( \frac{K}{1 + K} \right)^K \cdot A \cdot B,$$

where,

$$A = \left[ 1 - \frac{1 + 2K}{1 + K} \left( \frac{K}{1 + K} \right)^K \right],$$

and

$$B = \left[ (1 - C)^K - \left( \frac{1 - C + c - q}{1 - q} \right)^K \right].$$

Since $K > 0$, we have

$$\left( \frac{1 + K}{K} \right)^K \geq 1 + \frac{K}{K} = 2 \geq \frac{1 + 2K}{1 + K}.$$

Thus, we have $1 - \frac{1+2K}{1+K} \left( \frac{K}{1+K} \right)^K > 0$. Since $qC \geq c$, we have $(1 - q)(1 - C) \geq 1 - C + c - q$. Thus, we have $\Delta^{SC} \geq \Delta^{VI}$ when $\frac{c}{q} \leq C \leq 1 + c - q$. Finally, when $1 + c - q \leq C \leq 1$, we have

$$\Delta^{SC} - \Delta^{VI} = \frac{1}{K} \left[ \frac{K(1 - C)}{1 + K} \right]^{1+K} \left[ 1 - \frac{1 + 2K}{K} \left( \frac{K}{1 + K} \right)^{1+K} \right].$$

Since $K > 0$, we have

$$\left( \frac{1 + K}{K} \right)^{1+K} \geq 1 + \frac{1 + K}{K} = \frac{1 + 2K}{K}.$$

Thus, we have $\Delta^{SC} \geq \Delta^{VI}$ when $1 + c - q \leq C \leq 1$. Therefore, we have proved that $\Delta^{SC} \geq \Delta^{VI}$ for all cases. $\square$

**Derivation of Retailer's Optimal Promotional Effort:**

We use the following properties to prove Theorem 3.4.1. Denote $\pi^n = \pi(w, p_n, q)$, $\pi^p = \pi(w, 1, p_p)$, and $\pi^b = \pi(w, p_n, p_p)$.

**Property B.0.1.** When $(p_n, p_p) \in R_N$, the optimal values of $x$ and $y$ are given by the following:

1. If $\pi^n \geq \pi^p$, then $x = \frac{1}{a}\pi^n$ and $y = 0$. The retailer's profit is $\pi = \pi^n + \frac{1}{2a}(\pi^n)^2$.

2. If $\pi^n \leq \pi^p$, then $x = 0$ and $y = \frac{1}{a}\pi^p$. The retailer's profit is $\pi = \pi^n + \frac{1}{2a}(\pi^p)^2$.

*Proof.* If $\pi^n \geq \pi^p$, we have

$$\pi(x,y) - \pi(y,0) = (y-x)(\pi^p - \pi^n) - \frac{1}{2}rax^2 \leq 0.$$

when $0 \leq x \leq y$ and

$$\pi(x,y) - \pi(x,0) = (y-x)\pi^p - \frac{1}{2}(1+r)ax^2 - \frac{1}{2}ay^2 \leq 0.$$

when $0 \leq y \leq x$. Thus, we have $y = 0$. The retailer's profit can be written as $\pi = (1+x)\pi^n - \frac{1}{2}ax^2$ and the optimal value of $x$ is $x = \frac{1}{a}\pi^n$. Substituting into the profit, we have $\pi = \pi^n + \frac{1}{2a}(\pi^n)^2$. Similarly, we can prove the results for the case $\pi^n \leq \pi^p$. $\square$

**Property B.0.2.** When $(p_n, p_p) \in R_B$, the optimal values of $x$ and $y$ are given by the following:

1. When $\pi^n \geq \pi^p$:

   (a) If $\pi^n \geq \pi^b$, then $x = \frac{1}{a}\pi^n$ and $y = 0$. The retailer's profit is $\pi = \pi^b + \frac{1}{2a}(\pi^n)^2$.

(b) If $\pi^n \leq \pi^b \leq (1+r)\,\pi^n$, then $x = \frac{1}{a}\pi^n$ and $y = \frac{1}{ra}\left(\pi^b - \pi^n\right)$. The retailer's profit is $\pi = \pi^b + \frac{1}{2a}\left(\pi^n\right)^2 + \frac{1}{2ra}\left(\pi^b - \pi^n\right)^2$.

(c) If $\pi^b \geq (1+r)\,\pi^n$, then $x = y = \frac{1}{(1+r)a}\pi^b$. The retailer's profit is $\pi = \pi^b + \frac{1}{2(1+r)a}\left(\pi^b\right)^2$.

2. When $\pi^n \leq \pi^p$:

(a) If $\pi^p \geq \pi^b$, then $x = 0$ and $y = \frac{1}{a}\pi^p$. The retailer's profit is $\pi = \pi^b + \frac{1}{2a}\left(\pi^p\right)^2$.

(b) If $\pi^p \leq \pi^b \leq (1+r)\,\pi^p$, then $x = \frac{1}{ra}\left(\pi^b - \pi^p\right)$ and $y = \frac{1}{a}\pi^p$. The retailer's profit is $\pi = \pi^b + \frac{1}{2a}\left(\pi^p\right)^2 + \frac{1}{2ra}\left(\pi^b - \pi^p\right)^2$.

(c) If $\pi^b \geq (1+r)\,\pi^p$, then $x = y = \frac{1}{(1+r)a}\pi^b$. The retailer's profit is $\pi = \pi^b + \frac{1}{2(1+r)a}\left(\pi^b\right)^2$.

*Proof.* We first consider the case in which $\pi^n \geq \pi^p$. We show that we must have $0 \leq y \leq x$. Otherwise, we have

$$\pi\,(x, y) - \pi\,(y, x) = (y - x)\,(\pi^p - \pi^n) < 0.$$

Thus, the retailer's profit can be rewritten as $\pi = (1+y)\,\pi^b + (x-y)\,\pi^n - \frac{1}{2}ray^2 - \frac{1}{2}ax^2$. If we have $\pi^n \geq \pi^b$, then $\pi\,(x, y) - \pi\,(x, 0) = y\left(\pi^b - \pi^n\right) - \frac{1}{2}ray^2 \leq 0$. Thus, we have $y = 0$ and the retailer's profit is given by $\pi = \pi^b + x\pi^n - \frac{1}{2}ax^2$. The optimal value of $x$ is $x = \frac{1}{a}\pi^n$. Substituting into the profit function gives $\pi = \pi^b + \frac{1}{2a}\left(\pi^n\right)^2$. If $\pi^b \geq \pi^n$, the solution for the first order conditions in terms of $x$ is $x = \frac{1}{a}\pi^n$. Thus, if $y \leq \frac{1}{a}\pi^n$, we have $x = \frac{1}{a}\pi^n$

and the retailer's profit is given by $\pi = (1+y)\,\pi^b + \frac{1}{2a}\left(\pi^n\right)^2 - y\pi^n - \frac{1}{2}ray^2$. The solution for the first order conditions in terms of $y$ is $y = \frac{1}{ra}\left(\pi^b - \pi^n\right)$. The condition for $\frac{1}{ra}\left(\pi^b - \pi^n\right) \leq \frac{1}{a}\pi^n$ is equivalent to $\pi^b \leq (1+r)\,\pi^n$. Similarly, if $y \geq \frac{1}{a}\pi^n$, we have $x = y$ and the retailer's profit is $\pi = (1+y)\,\pi^b - \frac{1}{2}(1+r)\,ay^2$ and the solution for the first order conditions in terms of $y$ is given by $y = \frac{1}{(1+r)a}\pi^b$. The condition for $\frac{1}{(1+r)a}\pi^b \geq \frac{1}{a}\pi^n$ is equivalent to $\pi^b \geq (1+r)\,\pi^n$. Therefore, we have $x = \frac{1}{a}\pi^n$, $y = \frac{1}{ra}\left(\pi^b - \pi^n\right)$, and the retailer's profit is $\pi = \pi^b + \frac{1}{2a}\left(\pi^n\right)^2 + \frac{1}{2ra}\left(\pi^b - \pi^n\right)^2$ if $\pi^n \leq \pi^b \leq (1+r)\,\pi^n$; and $x = y = \frac{1}{(1+r)a}\pi^b$ and the retailer's profit is $\pi = \pi^b + \frac{1}{2(1+r)a}\left(\pi^b\right)^2$ if $\pi^b \geq (1+r)\,\pi^n$. Similarly, we can prove the results for the case of $\pi^n \leq \pi^p$. □

**Property B.0.3.** When $(p_n, p_p) \in R_P$, the optimal values of $x$ and $y$ are given by the following:

1. If $\pi^n \geq \pi^p$, then $x = \frac{1}{a}\pi^n$ and $y = 0$. The retailer's profit is $\pi = \pi^p + \frac{1}{2a}\left(\pi^n\right)^2$.

2. If $\pi^n \leq \pi^p$, then $x = 0$ and $y = \frac{1}{a}\pi^p$. The retailer's profit is $\pi = \pi^p + \frac{1}{2a}\left(\pi^p\right)^2$.

*Proof.* First consider the case $\pi^n \geq \pi^p$. If $0 \leq x \leq y$, we have $\pi(x,y) - \pi(y,0) = y\left(\pi^p - \pi^n\right) - \frac{1}{2}rax^2 \leq 0$. If $0 \leq y \leq x$, we have $\pi(x,y) - \pi(x,0) = y\left(\pi^p - \pi^n\right) - \frac{1}{2}ray^2 \leq 0$. Thus, the retailer sets $y = 0$. The retailer's profit can be rewritten as $\pi = \pi^p + x\pi^n - \frac{1}{2}ax^2$ and the optimal value of $x$ is $x = \frac{1}{a}\pi^n$. Substituting into the profit function, we have $\pi = \pi^p + \frac{1}{2a}\left(\pi^n\right)^2$. Similarly, we can prove the results for the case in which $\pi^p \geq \pi^n$. □

**Proof of Theorem 3.4.1:**

*Proof.* For notational simplicity, denote $\pi^n = \pi(w, p_n, q)$, $\pi^p = \pi(w, 1, p_p)$, and $\pi^b = \pi(w, p_n, p_p)$. Also denote $\hat{\pi}^n = \pi(w, p_n^1, q)$, $\hat{\pi}^b = \pi\left(w, p_n^1, p_p^1\right)$, and $\hat{\pi}^p = \pi\left(w, 1, p_p^1\right)$. The four constants for the wholesale price are defined as follows. First, we have $w_1 = \frac{c}{q}$ and $w_4 = 1 - q + c$. The condition $\hat{\pi}^n \geq \hat{\pi}^p$ is equivalent to $w \leq w_{n-p} = 1 - q^{1/(1+K)} + cq^{-K/(1+K)}$, where $\frac{c}{q} \leq w_{n-p} \leq 1 - q + c$. The constant $w_2$ can be found as follows. We prove that $(1+r)\hat{\pi}^n - \hat{\pi}^b$ is decreasing in $w$ when $\frac{c}{q} \leq w \leq 1 + c - q$. In fact, we have

$$(1+r)\frac{\partial \hat{\pi}^n}{\partial w} = -(1+r)\left[\frac{K(1-w)}{1+K}\right]^K < -\left[\frac{K(1+c-q-w)}{(1+K)(1-q)}\right]^K = -\frac{\partial \hat{\pi}^b}{\partial w},$$

when $0 < r < 1$ and $\frac{c}{q} \leq w \leq 1 - q + c$. Thus, if $(1+r)\hat{\pi}^n - \hat{\pi}^b \geq 0$ at $w = w_{n-p}$, we set $w_2 = w_{n-p}$. Otherwise, there is a unique solution for $(1+r)\hat{\pi}^n - \hat{\pi}^b = 0$ and we set that solution as $w_2$. Then we have $w_1 \leq w_2 \leq w_{n-p}$ and $\hat{\pi}^b \leq (1+r)\hat{\pi}^n$ when $w_1 \leq w \leq w_2$. Similarly, we can define the constant $w_3$, such that $w_{n-p} \leq w_3 \leq w_4$ and $\hat{\pi}^b \geq (1+r)\hat{\pi}^p$ when $w_{n-p} \leq w \leq w_3$.

Next, we identify the retailer's best response in each of the five intervals (some of them may be a single point) defined by the four constants. First, we consider the case $0 \leq w \leq w_1 = \frac{c}{q}$. When $p_n = p_n^1$, $x = \frac{1}{a}\hat{\pi}^n$, and $y = 0$, the retailer's optimal profit is given by $\pi^*(w) = \hat{\pi}^n + \frac{1}{2a}(\hat{\pi}^n)^2$. Now we prove that $\pi^* \geq \pi(w, p_n, p_p, x, y)$ for any values of $p_n$ and $p_p$, where $x$ and $y$ are given in Property B.0.1. We refer to $\pi^*(w)$ and $\pi(w, p_n, p_p, x, y)$ simply as $\pi^*$ and $\pi$ in the rest of the proof when there is no confusion.

Consider the case of $(p_n, p_p) \in R_N$. From Property B.0.1, if $\pi^n \geq \pi^p$, then $\pi = \pi^n + \frac{1}{2a}\left(\pi^n\right)^2 \leq \pi^*$ since $\pi$ is increasing in $\pi^n$ and $\pi^n \leq \hat{\pi}^n$. If $\pi^n \leq \pi^p$, then $\pi = \pi^n + \frac{1}{2a}\left(\pi^p\right)^2 \leq \pi^*$ since $\hat{\pi}^n \geq \pi^p$ for any values of $p_p$ from the proof of Lemma 3.3.2 and $\pi^n \leq \hat{\pi}^n$.

Next, consider the case of $(p_n, p_p) \in R_B$. From the proof of Lemma 3.3.2, we have $\hat{\pi}^n \geq \pi^b$ for any values of $(p_n, p_p) \in R_B$. From Property B.0.2, if $\pi^n \geq \pi^p$ and $\pi^n \geq \pi^b$, we have $\pi = \pi^b + \frac{1}{2a}\left(\pi^n\right)^2 \leq \pi^n + \frac{1}{2a}\left(\pi^n\right)^2 \leq \pi^*$. If $\pi^p \leq \pi^n \leq \pi^b \leq (1+r)\,\pi^n$, then $\pi = \pi^b + \frac{1}{2a}\left(\pi^n\right)^2 + \frac{\left(\pi^b - \pi^n\right)^2}{2ra}$. We have $\frac{\partial \pi}{\partial \pi^n} = \frac{(1+r)a\pi^n - a\pi^b}{ra^2} \geq 0$. Thus, $\pi \leq \pi^b + \frac{1}{2a}\left(\pi^b\right)^2 \leq \pi^*$ since $\frac{1}{1+r}\pi^b \leq \pi^n \leq \pi^b \leq \hat{\pi}^n$. If $\pi^p \leq \pi^n$ and $\pi^b \geq (1+r)\,\pi^n$, then $\pi = \pi^b + \frac{1}{2(1+r)a}\left(\pi^b\right)^2$. Since $\pi^b \leq \hat{\pi}^n$ and $0 < r < 1$, we have $\pi \leq \hat{\pi}^n + \frac{1}{2(1+r)a}\left(\hat{\pi}^n\right)^2 \leq \pi^*$. If $\pi^n, \pi^b \leq \pi^p$, then $\pi = \pi^b + \frac{1}{2a}\left(\pi^p\right)^2 \leq \pi^*$ since $\pi^b, \pi^p \leq \hat{\pi}^n$. If $\pi^n \leq \pi^p \leq \pi^b \leq (1+r)\,\pi^p$, then $\pi = \pi^b + \frac{1}{2a}\left(\pi^p\right)^2 + \frac{\left(\pi^b - \pi^p\right)^2}{2ra}$. Similarly we can check that $\pi$ is increasing in $\pi^p$ and thus $\pi \leq \pi^b + \frac{1}{2a}\left(\pi^b\right)^2 \leq \pi^*$ since $\pi^b \leq \hat{\pi}^n$.

In the case of $(p_n, p_p) \in R_P$, from Property B.0.3, if $\pi^n \geq \pi^p$, we have $\pi = \pi^p + \frac{1}{2a}\left(\pi^n\right)^2 \leq \pi^*$. If $\pi^n \leq \pi^p$, then $\pi = \pi^p + \frac{1}{2a}\left(\pi^p\right)^2 \leq \pi^*$. Thus, we have proved the first result.

Similarly, we can prove the results for other values of $w$. $\square$

# Appendix C

# Proofs for Chapter 4

**Proof of Lemma 4.3.1:**

*Proof.* The incumbent's profit function is given by $\pi_I(q_H, q_{IL}, q_{EL}) = [p_H(q_H, q_{IL} + q_{EL}) - c - c_H]$ $[p_L(q_H, q_{IL} + q_{EL}) - c]\, q_{IL}$, where, $q_H \geq 0$ and $q_{IL} \geq 0$. The profit maximizing $q_H$ and $q_{IL}$ are

$$q_H = \begin{cases} \frac{1}{2}(1 - c - c_H - \gamma q_{EL} - 2\gamma q_{IL}), & \text{if } q_{EL} + 2q_{IL} \leq \frac{1-c-c_H}{\gamma}; \\ 0, & \text{otherwise.} \end{cases}$$

and

$$q_{IL} = \begin{cases} \frac{1}{2}\left(1 - \frac{c}{\gamma} - 2q_H - q_{EL}\right), & \text{if } 2q_H + q_{EL} \leq 1 - \frac{c}{\gamma}; \\ 0, & \text{otherwise.} \end{cases}$$

Similarly, the entrant solves the problem

$$\max_{q_{EL} \geq 0} \pi_E(q_H, q_{IL}, q_{EL}) = [p_L(q_H, q_{IL} + q_{EL}) - c]\, q_{EL},$$

for $q_{EL} \geq 0$ and enters the market whenever $q_{EL} > 0$. The solution is given by

$$q_{EL} = \begin{cases} \frac{1}{2}\left(1 - \frac{c}{\gamma} - q_H - q_{IL}\right), & \text{if } q_H + q_{IL} \leq 1 - \frac{c}{\gamma}; \\ 0, & \text{otherwise.} \end{cases}$$

There are eight different combinations for the three quantities. To find the equilibrium quantities for both the incumbent and the entrant, we solve the equations in each of the eight combinations and check if the conditions are all satisfied. We omit the details in the paper. $\square$

**Proof of Proposition 4.3.2:**

*Proof.* When $c_H \in \left[0, \max\left\{0, \frac{2c-\gamma-c\gamma}{\gamma}\right\}\right]$, we have $\pi_I^{NNN} - \pi_I^{NNR} = 0$, $q_H^{NNN} - q_H^{NNR} = 0$, and $q_{IL}^{NNN} - q_{IL}^{NNR} = 0$. The results are trivial. When $c_H \in \left[\max\left\{0, \frac{2c-\gamma-c\gamma}{\gamma}\right\}, \frac{c-c\gamma}{\gamma}\right]$, we have

$$\begin{aligned}
q_H^{NNN} - q_H^{NNR} &= \frac{\gamma\left(1+c+c_H\right)-2c}{2\left(4-\gamma\right)} = \frac{\gamma}{2\left(4-\gamma\right)}\left(c_H - \frac{2c-\gamma-c\gamma}{\gamma}\right) \geq 0, \\
\pi_I^{NNN} - \pi_I^{NNR} &= \left(q_H^{NNN}\right)^2 - \left(q_H^{NNR}\right)^2 \geq 0 \\
q_{IL}^{NNN} - q_{IL}^{NNR} &= 0.
\end{aligned}$$

Thus, we have proved the results. $\square$

**Definition of Regions $R_{NCR}^i$, $i = 1, 2, 3, 4$:**

The four regions $R_{NCR}^i$ $(i = 1, 2, 3, 4)$ for the values of $(c,\, c_H)$ in lemma 4.3.3 are defined as follows:

$$\begin{aligned}
R_{NCR}^1 &\equiv \left\{0 < c \leq \frac{\gamma^2}{4-3\gamma} \textit{ and } 0 \leq c_H < \frac{c-c\gamma}{\gamma}\right\} \cup \\
&\quad \left\{\frac{\gamma^2}{4-3\gamma} < c \leq \frac{\gamma}{2-\gamma} \textit{ and } 0 \leq c_H \leq \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}\right\}, \\
R_{NCR}^2 &\equiv \left\{\frac{\gamma^2}{4-3\gamma} < c \leq \frac{\gamma}{2-\gamma} \textit{ and } \frac{2c-c\gamma-\gamma^2}{8-2\gamma} \leq c_H < \frac{c-c\gamma}{\gamma}\right\} \cup \\
&\quad \left\{\frac{\gamma}{2-\gamma} < c \leq \gamma \textit{ and } \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma} \leq c_H < \frac{c-c\gamma}{\gamma}\right\}, \\
R_{NCR}^3 &\equiv \left\{\frac{\gamma}{2-\gamma} < c \leq \gamma \textit{ and } 0 \leq c_H \leq \frac{2c-\gamma-c\gamma}{\gamma}\right\}, \\
R_{NCR}^4 &\equiv \left\{\frac{\gamma}{2-\gamma} < c \leq \gamma \textit{ and } \frac{2c-\gamma-c\gamma}{\gamma} \leq c_H \leq \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}\right\}.
\end{aligned}$$

**Proof of Lemma 4.3.3:**

*Proof.* Relaxing the constraint $q_{EL} \geq 0$ and solve the first order condition for the entrant, we have $q_{EL} = \left[\gamma\left(1 - q_H - q_{IL}\right) - c\right]/\left(2\gamma\right)$. When $\gamma\left(1 - q_H - q_{IL}\right) - c \geq 0$, we have $q_{EL} = \left[\gamma\left(1 - q_H - q_{IL}\right) - c\right]/\left(2\gamma\right)$; otherwise, $q_{EL} = 0$. Thus, we have the rival's best response as follows:

$$q_{EL}\left(q_H, q_{IL}\right) = \begin{cases} \frac{\gamma(1-q_H-q_{IL})-c}{2\gamma}, & if\ q_H + q_{IL} \leq \frac{\gamma-c}{\gamma}; \\ 0, & if\ q_H + q_{IL} \geq \frac{\gamma-c}{\gamma}. \end{cases} \tag{C.1}$$

Similarly, relaxing the constraints $0 \leq q_H \leq K$ and $q_{IL} \geq 0$ and solve the first order conditions for $q_H$ and $q_{IL}$, respectively, we have $q_H = \frac{1}{2}\left(1 - c - \gamma q_{EL} - 2\gamma q_{IL}\right)$ and $q_{IL} = \frac{1}{2}\left(1 - q_{EL} - 2q_H - \frac{c}{\gamma}\right)$. Thus, we have

$$q_H = \begin{cases} 0, & \text{if } q_{EL} + 2q_{IL} \geq \frac{1-c}{\gamma}; \\ \frac{1}{2}\left(1 - c - \gamma q_{EL} - 2\gamma q_{IL}\right), & \text{if } \frac{1-c-2K}{\gamma} \leq q_{EL} + 2q_{IL} \leq \frac{1-c}{\gamma} \\ K, & \text{if } q_{EL} + 2q_{IL} \leq \frac{1-c-2K}{\gamma}. \end{cases} \tag{C.2}$$

and

$$q_{IL} = \begin{cases} 0, & q_{EL} + 2q_H \geq \frac{\gamma-c}{\gamma}; \\ \frac{1}{2}\left(1 - q_{EL} - 2q_H - \frac{c}{\gamma}\right), & q_{EL} + 2q_H \leq \frac{\gamma-c}{\gamma}. \end{cases} \tag{C.3}$$

Thus, given the incumbent's capacity $K$ for product H, the best response sales quantities for both the incumbent and the entrant are given by:

$$\begin{cases} q_H = K, & q_{IL} = \frac{\gamma-c}{3\gamma} - K, & q_{EL} = \frac{\gamma-c}{3\gamma}, & 0 < c < \gamma,\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ 0 \leq K \leq \frac{\gamma-}{3\gamma} \\ q_H = K, & q_{IL} = 0, & q_{EL} = \frac{1}{2}\left(\frac{\gamma-c}{\gamma} - K\right), & 0 < c \leq \frac{\gamma}{2-\gamma},\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ \frac{\gamma-c}{3\gamma} \leq K \\ & & & \frac{\gamma}{2-\gamma} < c < \gamma,\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ \frac{\gamma-c}{3\gamma} \leq K \\ q_H = \frac{2-\gamma-c}{4-\gamma}, & q_{IL} = 0, & q_{EL} = \frac{\gamma-c(2-\gamma)}{\gamma(4-\gamma)}, & 0 < c \leq \frac{\gamma}{2-\gamma},\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ K \geq \frac{2-\gamma-}{4-\gamma} \\ q_H = K, & q_{IL} = 0, & q_{EL} = 0, & \frac{\gamma}{2-\gamma} < c < \gamma,\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ \frac{\gamma-c}{\gamma} \leq K \\ q_H = \frac{1-c}{2}, & q_{IL} = 0, & q_{EL} = 0, & \frac{\gamma}{2-\gamma} < c < \gamma,\ 0 \leq c_H < \frac{c(1-\gamma)}{\gamma},\ K \geq \frac{1-c}{2}. \end{cases} \tag{C.4}$$

Based on the best responses of the incumbent and the rival, the optimal capacity level $K$ of the high end product for the incumbent can be found as follows. When $0 < \gamma < 1$, $0 < c \leq \gamma/(2-\gamma)$, and $0 \leq c_H < \frac{c(1-\gamma)}{\gamma}$, the incumbent's profit as a function of $K$ is given as follows:

$$\pi_I^{NCR-1} = \begin{cases} \pi_{I,1}^{NCR-1} = \frac{(\gamma-c)^2}{9\gamma} + K\left[(1-\gamma)(1-K) - c_H\right], & if\ 0 \leq K \leq \frac{\gamma-c}{3\gamma}; \\ \pi_{I,2}^{NCR-1} = \frac{1}{2}K\left[(2-\gamma)(1-K) - c - 2c_H\right], & if\ \frac{\gamma-c}{3\gamma} \leq K \leq \frac{2-\gamma-c}{4-\gamma}; \\ \pi_{I,3}^{NCR-1} = \left(\frac{2-c-\gamma}{4-\gamma}\right)^2 - c_H K, & if\ K \geq \frac{2-\gamma-c}{4-\gamma}. \end{cases}$$

It is a continous function of $K$ and decreasing in the third branch. Solving the first order conditions for $\pi_{I,1}^{NCR-1}$ and $\pi_{I,2}^{NCR-1}$, we have $K_1^{NCR-1} = \frac{1}{2} - \frac{c_H}{2(1-\gamma)}$ and $K_2^{NCR-1} = \frac{1}{2} - \frac{c+2c_H}{2(2-\gamma)}$. The optimal value of $K$ can be characterized as follows:

$$K^{NCR-1} = \begin{cases} K_1^{NCR-1}, & if\ K_1^{NCR-1} \leq \frac{s-c}{3s}; \\ \frac{s-c}{3s}, & if\ K_2^{NCR-1} \leq \frac{s-c}{3s} \leq K_1^{NCR-1}; \\ K_2^{NCR-1}, & if\ \frac{s-c}{3s} \leq K_2^{NCR-1} \leq \frac{2-s-c}{4-s}; \\ \frac{2-s-c}{4-s}, & if\ K_2^{NCR-1} \geq \frac{2-s-c}{4-s}. \end{cases}$$

The conditions are simplified as follows:

$$K^{NCR-1} = \begin{cases} \frac{1}{2} - \frac{c+2c_H}{2(2-\gamma)}, & if\ 0 < \gamma < 1,\ \frac{\gamma^2}{4-3\gamma} < c \leq \frac{\gamma}{2-\gamma},\ and\ \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma} \leq c_H \leq \frac{c-c\gamma}{\gamma}; \\ \frac{2-\gamma-c}{4-\gamma}, & if\ 0 < \gamma < 1,\ 0 < c \leq \frac{\gamma^2}{4-3\gamma},\ and\ 0 \leq c_H < \frac{c-c\gamma}{\gamma};\ or \\ & if\ 0 < \gamma < 1,\ \frac{\gamma^2}{4-3\gamma} < c \leq \frac{\gamma}{2-\gamma},\ and\ 0 \leq c_H \leq \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}. \end{cases}$$

When $0 < \gamma < 1$, $\gamma/(2-\gamma) < c < \gamma$, and $0 \leq c_H \leq c(1-\gamma)/\gamma$, the incumbent's profit as a function of $K$ is given as follows:

$$\pi_I^{NCR-2} = \begin{cases} \pi_{I,1}^{NCR-2} = \frac{(\gamma-c)^2}{9\gamma} + K\left[(1-\gamma)(1-K) - c_H\right], & if\ 0 \leq K \leq \frac{\gamma-c}{3\gamma}; \\ \pi_{I,2}^{NCR-2} = \frac{1}{2}K\left[(2-\gamma)(1-K) - c - 2c_H\right], & if\ \frac{\gamma-c}{3\gamma} \leq K \leq \frac{\gamma-c}{\gamma}; \\ \pi_{I,3}^{NCR-2} = K(1-c-c_H-K), & if\ \frac{\gamma-c}{\gamma} \leq K \leq \frac{1-c}{2}; \\ \pi_{I,4}^{NCR-2} = \left(\frac{1-c}{2}\right)^2 - c_H K, & if\ K \geq \frac{1-c}{2}. \end{cases}$$

It is easy to check that it is continous in $K$ and decreasing in the fourth branch. The solutions for the first order conditions for each of the first three branchs are given by $K_1^{NCR-2} = \frac{1}{2} - \frac{c_H}{2(1-\gamma)}$, $K_2^{NCR-2} = \frac{1}{2} - \frac{c+2c_H}{2(2-\gamma)}$, and $K_3^{NCR-2} = \frac{1}{2}(1-c-c_H)$. Comparing the three maximizers, we have $K_1^{NCR-2} \geq K_2^{NCR-2} \geq K_3^{NCR-2}$. Thus, the optimal value of $K$ can be characterized as follows:

$$K^{NCR-2} = \begin{cases} K_1^{NCR-2}, & if\ K_1^{NCR-2} \leq \frac{\gamma-c}{3\gamma}; \\ \frac{\gamma-c}{3\gamma}, & if\ K_2^{NCR-2} \leq \frac{\gamma-c}{3\gamma} \leq K_1^{NCR-2}; \\ K_2^{NCR-2}, & if\ \frac{\gamma-c}{3\gamma} \leq K_2^{NCR-2} \leq \frac{\gamma-c}{\gamma}; \\ \frac{\gamma-c}{\gamma}, & if\ K_3^{NCR-2} \leq \frac{\gamma-c}{\gamma} \leq K_2^{NCR-2}; \\ K_3^{NCR-2}, & if\ \frac{\gamma-c}{\gamma} \leq K_3^{NCR-2} \leq \frac{1-c}{2}; \\ \frac{1-c}{2}, & if\ K_3^{NCR-2} \geq \frac{1-c}{2}. \end{cases}$$

Thus, we have

$$K^{NCR-2} = \begin{cases} K_2^{NCR-2}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c<\gamma,\ and\ \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma} \leq c_H < \frac{c-c\gamma}{\gamma}; \\ \frac{\gamma-c}{\gamma}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c<\gamma,\ and\ \frac{2c-\gamma-\gamma c}{\gamma} \leq c_H \leq \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}; \\ K_3^{NCR-2}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c<\gamma,\ and\ 0 \leq c_H \leq \frac{2c-\gamma-c\gamma}{\gamma}. \end{cases}$$

To summarize, we have the following optimal capacity level $K$ for the incumbent:

$$K^{NCR} = \begin{cases} \frac{2-\gamma-c}{4-\gamma}, & if\ 0<\gamma<1,\ 0<c\leq\frac{\gamma^2}{4-3\gamma},\ and\ 0\leq c_H<\frac{c-c\gamma}{\gamma},\ or \\ & if\ 0<\gamma<1, \frac{\gamma^2}{4-3\gamma}<c\leq\frac{\gamma}{2-\gamma},\ and\ 0\leq c_H\leq\frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}; \\ \frac{2-\gamma-c-2c_H}{4-2\gamma}, & if\ 0<\gamma<1,\ \frac{\gamma^2}{4-3\gamma}<c\leq\frac{\gamma}{2-\gamma},\ and\ \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}\leq c_H<\frac{c-c\gamma}{\gamma},\ or \\ & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}\leq c_H<\frac{c-c\gamma}{\gamma}; \\ \frac{1-c-c_H}{2}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ 0\leq c_H\leq\frac{2c-\gamma-c\gamma}{\gamma}; \\ \frac{\gamma-c}{\gamma}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ \frac{2c-\gamma-c\gamma}{\gamma}\leq c_H\leq\frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}. \end{cases} \tag{C.5}$$

□

**Proof of Proposition 4.3.4:**

*Proof.* The incumbent's production quantity $q_H^{NNR}$ for the high end product in model NNR and capacity level $K^{NCR}$ for the high end product in model NCR can be rewritten as follows:

| $c$ | $c_H$ | $q_H^{NNR}$ | | $K^{NCR}$ |
|---|---|---|---|---|
| $0 < c \leq \frac{\gamma^2}{4-3\gamma}$ | $0 \leq c_H < \frac{c-c\gamma}{\gamma}$ | $\frac{2-\gamma-c-2c_H}{4-\gamma}$ | $\leq$ | $\frac{2-\gamma-c}{4-\gamma}$ |
| $\frac{\gamma^2}{4-3\gamma} < c \leq \frac{\gamma}{2-\gamma}$ | $0 \leq c_H \leq \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}$ | $\frac{2-\gamma-c-2c_H}{4-\gamma}$ | $\leq$ | $\frac{2-\gamma-c}{4-\gamma}$ |
| | $\frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma} \leq c_H < \frac{c-c\gamma}{\gamma}$ | $\frac{2-\gamma-c-2c_H}{4-\gamma}$ | $\leq$ | $\frac{2-\gamma-c-2c_H}{4-2\gamma}$ |
| $\frac{\gamma}{2-\gamma} < c \leq \gamma$ | $0 \leq c_H \leq \frac{2c-\gamma-c\gamma}{\gamma}$ | $\frac{1-c-c_H}{2}$ | $=$ | $\frac{1-c-c_H}{2}$ |
| | $\frac{2c-\gamma-c\gamma}{\gamma} \leq c_H \leq \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}$ | $\frac{2-\gamma-c-2c_H}{4-\gamma}$ | $\leq$ | $\frac{\gamma-c}{\gamma}$ |
| | $\frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma} \leq c_H < \frac{c-c\gamma}{\gamma}$ | $\frac{2-\gamma-c-2c_H}{4-\gamma}$ | $\leq$ | $\frac{2-\gamma-c-2c_H}{4-2\gamma}$ |

Thus, we always have $K^{NCR} \geq q_H^{NNR}$.

The incumbent's profit in model NCR is given by:

$$\pi_I^{NCR} = \begin{cases} \frac{2-\gamma-c}{4-\gamma}\left(\frac{2-\gamma-c}{4-\gamma} - c_H\right), & if\ 0<\gamma<1,\ 0<c\leq\frac{\gamma^2}{4-3\gamma},\ and\ 0\leq c_H<\frac{c-c\gamma}{\gamma},\ or \\ & if\ 0<\gamma<1, \frac{\gamma^2}{4-3\gamma}<c\leq\frac{\gamma}{2-\gamma},\ and\ 0\leq c_H\leq\frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}; \\ \frac{(2-\gamma-c-2c_H)^2}{8(2-\gamma)}, & if\ 0<\gamma<1,\ \frac{\gamma^2}{4-3\gamma}<c\leq\frac{\gamma}{2-\gamma},\ and\ \frac{2\gamma-c\gamma-\gamma^2}{8-2\gamma}\leq c_H<\frac{c-c\gamma}{\gamma},\ or \\ & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ \frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}\leq c_H<\frac{c-c\gamma}{\gamma}; \\ \frac{(1-c-c_H)^2}{4}, & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ 0\leq c_H\leq\frac{2c-\gamma-c\gamma}{\gamma}; \\ \frac{\gamma-c}{\gamma}\left(\frac{c}{\gamma}-c-c_H\right), & if\ 0<\gamma<1,\ \frac{\gamma}{2-\gamma}<c\leq\gamma,\ and\ \frac{2c-\gamma-c\gamma}{\gamma}\leq c_H\leq\frac{4c-2\gamma-3c\gamma+\gamma^2}{2\gamma}. \end{cases}$$

It is easy to check that $\pi_I^{NCR} \leq \pi_I^{NNN}$. Obviously, we have $\pi_I^{NCR} \geq \pi_I^{NNR}$ since the incumbent has more options in model NCR than in model NNR. Therefore, we have both $\pi_I^{NCR} \leq \pi_I^{NNN}$ and $\pi_I^{NNR} \leq \pi_I^{NNN}$. $\square$

**Definition of Regions $R^i_{SNR}$, $i = 1, 2$:**

The two regions $R^i_{SNR}$ $(i = 1, 2)$ for the values of $(\gamma,\, c,\, c_H)$ in lemma 4.4.1 are defined as follows:

$$R^1_{SNR} \equiv \left\{2-\sqrt{2}<\gamma<1,\ 0<c\leq\frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma},\ and\ 0\leq c_H<\frac{c-c\gamma}{\gamma}\right\}\cup$$
$$\left\{\begin{array}{l} 2-\sqrt{2}<\gamma<1,\ \frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma}<c<\gamma,\ and \\ \frac{2-\gamma-c}{2}-\frac{1}{2}\sqrt{\frac{4c^2-8c\gamma-c^2\gamma+4\gamma^2+2c\gamma^2-\gamma^3}{\gamma}}\leq c_H<\frac{c-c\gamma}{\gamma} \end{array}\right\},$$
$$R^2_{SNR} = \left\{0<\gamma<1,\ 0<c<\gamma,\ 0\leq c_H<\frac{c\,(1-\gamma)}{\gamma}\right\}\backslash R^1_{SNR}.$$

**Proof of Lemma 4.4.1:**

*Proof.* The supplier's profit is given as in equation (4.7). Let $w_1^{SNR}$ and $w_2^{SNR}$ denote the solution for the first order conditions when $q_H(w)$, $q_{IL}(w)$, and $q_{EL}(w)$ take the values from the upper and lower branches in equation (**??**), respectively. Then, we have

$$w_1^{SNR}=\frac{\gamma\,(3-\gamma-c_H)+2c}{4},\ w_2^{SNR}=\frac{1+c-c_H}{2}.$$

The supplier's profit is concave in $w$ in both the upper and lower branches. However, we have $w_1^{SNR}\leq w_2^{SNR}$. Thus, the supplier's profit is not concave in $w$. The optimal wholesale price $w$ can be found as follows:

$$w^{SNR}=\begin{cases} w_1^{SNR}, & \text{if } w_2^{SNR}\leq\frac{\gamma(1+c_H)}{2-\gamma},\ or\ w_1^{SNR}\leq\frac{\gamma(1+c_H)}{2-\gamma}\leq w_2^{SNR}\ and\ \pi^{SNR}_{S,1}\left(w_1^{SNR}\right)\geq\pi^{SNR}_{S,2}\left(w_2^{SNR}\right) \\ w_2^{SNR}, & \text{if } w_1^{SNR}\geq\frac{\gamma(1+c_H)}{2-\gamma},\ or\ w_1^{SNR}\leq\frac{\gamma(1+c_H)}{2-\gamma}\leq w_2^{SNR}\ and\ \pi^{SNR}_{S,1}\left(w_1^{SNR}\right)\leq\pi^{SNR}_{S,2}\left(w_2^{SNR}\right) \end{cases}$$

where $\pi^{SNR}_{S,i}$ represents the supplier's profit if the wholesale price is given by $w_i^{SNR}$, $i = 1, 2$. The conditions are simplified as specified in the lemma. □

**Definition of Region $R_{SNR}$:**

The region $R_{SNR}$ for the values of ($\gamma$, $c$, $c_H$) in proposition 4.4.2 is defined as follows:

$$
\begin{aligned}
R_{SNR} \quad \equiv \quad & \left\{ \begin{array}{l} 2-\sqrt{2}<\gamma\leq 3-\sqrt{5},\ 0<c\leq \frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma},\ and \\ 0\leq c_H<\frac{c-c\gamma}{\gamma} \end{array} \right\} \cup \\
& \left\{ \begin{array}{l} 2-\sqrt{2}<\gamma\leq 3-\sqrt{5},\ \frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma}<c<\gamma,\ and \\ \frac{2-\gamma-c}{2}-\frac{1}{2}\sqrt{\frac{4c^2-8c\gamma-c^2\gamma+4\gamma^2+2c\gamma^2-\gamma^3}{\gamma}}\leq c_H<\frac{c-c\gamma}{\gamma} \end{array} \right\} \cup \\
& \left\{ \begin{array}{l} 3-\sqrt{5}<\gamma<1,\ \frac{6\gamma-4-\gamma^2}{2-\gamma}<c<\frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma},\ and \\ 0\leq c_H\leq \frac{4+2c-6\gamma-c\gamma+\gamma^2}{4} \end{array} \right\} \cup \\
& \left\{ \begin{array}{l} 3-\sqrt{5}<\gamma<1,\ \frac{\gamma}{2-\gamma}-\frac{\sqrt{4\gamma-9\gamma^2+6\gamma^3-\gamma^4}}{2-\gamma}<c<\gamma,\ and \\ \frac{2-\gamma-c}{2}-\frac{1}{2}\sqrt{\frac{4c^2-8c\gamma-c^2\gamma+4\gamma^2+2c\gamma^2-\gamma^3}{\gamma}}\leq c_H\leq \frac{4+2c-6\gamma-c\gamma+\gamma^2}{4} \end{array} \right\}.
\end{aligned}
$$

**Proof of Proposition 4.4.2:**

*Proof.* From theorem 4.4.1, the entrant has nonnegative demand when the supplier sets the wholesale price as $w = w_1^{SNR}$. In this case, the incumbent's profit is given by

$$\pi_{I,1}^{SNR} = \frac{\left[8-2c-(8-\gamma)\,c_H-(7-\gamma)\,\gamma\right]^2}{16\,(4-\gamma)^2}.$$

When the supplier sets the wholesale price as $w = w_2^{SNR}$, the incumbent's profit is the same as that in model SNN. Thus, we only need to compare the incumbent's profit in the case when $w = w_1^{SNR}$ to that in model SNN. Let $\pi_{I,1}^{SNR} \geq \pi_I^{SNN}$, together with the condition under which $w = w_1^{SNR}$, we have the conditions as specified in the proposition.

For the wholesale price, first, we have $w_2^{SNR} = w^{SNN}$. Thus, we only need to compare $w_1^{SNR}$ with $w^{SNN}$. We have

$$w^{SNN} - w_1^{SNR} = \frac{(2-\gamma)(1-\gamma-c_H)}{4}.$$

Since we have $0 < \gamma < 1$ and $0 \leq c_H < \frac{c-c\gamma}{\gamma} < 1-\gamma$, we always have $w^{SNN} > w_1^{SNR}$. Therefore, we prove the result. $\square$

**Proof of Lemma 4.4.3:**

*Proof.* From equation (4.8), we can find the first order solutions for wholesale price $w$ in each of the three intervals. Denote the solutions as $w_1^{SCN}(K)$, $w_2^{SCN}(K)$, and $w_3^{SCN}(K)$, respectively. We thus have $w_1^{SCN}(K) = \frac{\gamma+c}{2}$, $w_2^{SCN}(K) = 1-2K$, and $w_3^{SCN}(K) = \frac{1+c}{2}$. By checking the conditions for each first order solution to be the global optimal solution, we have the results as stated in the lemma. $\square$

**Proof of Lemma 4.4.4:**

*Proof.* From Lemma 4.4.3, we know that the incumbent's profit is concave in $K$ when $0 \leq K \leq \frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}}$, convext in $K$ when $\frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}} < K \leq \frac{1-c}{4}$, and strictly decreasing in $K$ when $K > \frac{1-c}{4}$. Furthermore, we know that the incumbent's profit is not continuous at $K = \frac{1-c}{4} - \frac{1}{4}\sqrt{\frac{(1-\gamma)(\gamma-c^2)}{\gamma}}$. By comparing the conditions for each possible global optimizers, we have the results as stated in the lemma. $\square$

**Proof of Proposition 4.4.5:**

*Proof.* The proof comes here... ☐

# Appendix D

# Proofs for Chapter 5

**Proof of Lemma 5.3.1:**

*Proof.* The results are straightforward from the expression for $w_n$ given in equation 5.6. $\square$

**Proof of Lemma 5.3.2:**

*Proof.* From Table 5.3, we have $w_p^1 = \frac{(1+b)(1-\gamma)\gamma}{4+2b(2-\gamma)}$. It is easy to check that $w_p^1$ is concave in $\gamma$ in the interval $0 < \gamma \leq \frac{1+b}{1+2b}$. At $\gamma = \frac{1+b}{1+2b}$, we have

$$\frac{\partial w_p^1}{\partial \gamma} = \frac{-2 + (-3+b)\, b}{2\,(2+3b)^2}.$$

When $b > \frac{1}{2}\left(3+\sqrt{17}\right)$, we have $\partial w_p^1/\partial\gamma \geq 0$. Thus, $w_p^1$ is increasing in $\gamma$ in $0 < \gamma \leq \frac{1+b}{1+2b}$. When $0 < b < \frac{1}{2}\left(3+\sqrt{17}\right)$, we have $\partial w_p^1/\partial\gamma < 0$ at $\gamma = \frac{1+b}{1+2b}$. Solve $\partial w_p^1/\partial\gamma = 0$ for $\gamma$ in $0 < \gamma \leq \frac{1+b}{1+2b}$, we have $\gamma = \frac{1}{b}\left(2+2b-\sqrt{2\,(1+b)\,(2+b)}\right)$. Therefore, we have proved the results. $\square$

**Proof of Lemma 5.3.3:**

*Proof.* From Table 5.3, we have

$$\frac{\partial \pi_A^1}{\partial \gamma} = \frac{(1+b)\,(2+b)\,(2\,(1+b)\,(3+b) - b\,(7+3b)\,\gamma)}{8\,(2+b\,(2-\gamma))^3}.$$

The denominator is positive for $0 < \gamma \leq \frac{1+b}{1+2b}$. The numerator is decreasing in $\gamma$. Thus, it is sufficient to show that the numerator is positive at $\gamma = \frac{1+b}{1+2b}$. Substituting $\gamma = \frac{1+b}{1+2b}$ into the numerator, we get $(1+b)^2(2+b)(6+b) > 0$. Thus, we have shown that $\pi_A^1$ is increasing in $\gamma$.

Taking the second derivative of $\pi_A^1$ with respect to $b$, we have

$$\frac{\partial^2 \pi_A^1}{\partial b^2} = \frac{(1-\gamma)\gamma(2(4+\gamma)+b(8-\gamma(8-5\gamma)))}{4(2+b(2-\gamma))^4} \geq 0.$$

Thus, $\pi_A^1$ is convex in $b$.

Therefore, we have proved the results. □

**Proof of Proposition 5.3.4:**

*Proof.* Denote $w_n^i$ as the equilibrium wholesale price for the national brand in sub-game $e = i$, $i = 0, 1$. Then we have

$$w_n^0 = \frac{(1+b)(1-\gamma)}{2+2b(1-\gamma)}, \quad and \quad w_n^1 = \frac{(1+b)(1-\gamma)}{2+b(2-\gamma)}.$$

Taking the difference, we have

$$w_n^0 - w_n^1 = \frac{(1+b)(1-\gamma)b\gamma}{2(2+2b-b\gamma)(1+b-b\gamma)} > 0,$$

since we have $0 < \gamma < 1$ and $b > 0$. □

**Proof of Lemma 5.3.5:**

*Proof.* As specified in Table 5.2 and Table 5.3, the supplier's profits satisfy the following:

$$\Pi_S^0 - \Pi_S^1 = \frac{(1+b)^2(1-\gamma)}{8+8b(1-\gamma)} - \frac{(1+b)^2(1-\gamma)}{8+4b(2-\gamma)} > 0$$

since $2(1-\gamma) < 2-\gamma$ for $0 < \gamma < 1$. To prove $\pi_A^1 \geq \pi_A^0$, we use the property that $\pi_A^1 - \pi_A^0$ is concave in $\gamma$ from Lemma 5.3.8. It is sufficient to prove that $\pi_A^1 - \pi_A^0 \geq 0$ at both $\gamma = 0$ and $\gamma = \frac{1+b}{1+2b}$. It is easy the check that $\pi_A^1 - \pi_A^0 = 0$ at $\gamma = 0$. When $\gamma = \frac{1+b}{1+2b}$, we have

$$\pi_A^1 - \pi_A^0 = \frac{b^2 (6 + b (13 + 2b (4 + b)))}{16 (1 + b)^2 (2 + 3b)^2} \geq 0.$$

Thus, we have $\pi_A^1 - \pi_A^0 \geq 0$. For the supply chain, we have

$$\Pi_{SC}^1 - \Pi_{SC}^0 = \frac{b (1 + b)^2 (1 - \gamma) \gamma (3 + b (3 - 2\gamma))}{16 (2 + b (2 - \gamma))^2 (1 + b - b\gamma)} \geq 0.$$

Similarly, for retailer B, we can show that $\pi_B^1 - \pi_B^0$ is concave in $\gamma$. At $\gamma = 0$, we have $\pi_B^1 - \pi_B^0 = 0$; while

$$\pi_B^1 - \pi_B^0 = \frac{b^2 (1 + b (1 + b) (11 + 8b))}{16 (1 + b)^2 (2 + 3b)^2} \geq 0$$

when $\gamma = \frac{1+b}{1+2b}$. Thus, we have $\pi_B^1 - \pi_B^0 \geq 0$. We have proved the results in item 1.

For result 2, we have

$$\pi_{A,p}^1 - \pi_{A,p}^0 = \frac{(1 + b) \gamma (4 + b (5 + b - (3 + b) \gamma))}{8 (2 + b (2 - \gamma))^2} - \frac{(1 + b) \gamma}{8 + 8b (1 - \gamma)}.$$

We can show that $\pi_{A,p}^1 - \pi_{A,p}^0$ is concave in $\gamma$ when $0 < \gamma \leq \frac{1+b}{1+2b}$. Solve $\pi_{A,p}^1 - \pi_{A,p}^0 = 0$ for $\gamma$, we have $\gamma = 0$ or $\gamma = \frac{(1+b)\left(2+2b-\sqrt{9+4b}\right)}{2b(2+b)}$ in the interval $0 < \gamma \leq \frac{1+b}{1+2b}$. Therefore, we have proved result 2. $\square$

**Proof of Proposition 5.3.7:**

*Proof.* From the definition of equilibrium, the proof is trivial. $\square$

**Proof of Lemma 5.3.8:**

*Proof.* The concavity of $\Delta\pi_A$ in $\gamma$ is built by showing that the second derivative of $\Delta\pi_A$ with respect to $\gamma$ is nonpositive. Since $\Delta\pi_A$ is concave in $\gamma$, $\Delta\pi_A$ will be increasing in $\gamma$ if $\partial\Delta\pi_A/\partial\gamma \geq 0$ at $\gamma = \frac{1+b}{1+2b}$. Substituting $\gamma = \frac{1+b}{1+2b}$ into $\partial\Delta\pi_A/\partial\gamma$, we have

$$\left.\frac{\partial\Delta\pi_A}{\partial\gamma}\right|_{\gamma=\frac{1+b}{1+2b}} = \frac{b\,(1+2b)^2\,(-12+b\,(-32+b\,(-25+b\,(-8+b\,(-3+2b)))))}{16\,(1+b)^4\,(2+3b)^3}.$$

The term in the second paranthesis of the numerator is increasing in $b$. It is negative at $b=0$. While it goes to infinity when $b$ goes to infinity. Thus, there exists a unique value $b_A^1 > 0$, such that $\partial\Delta\pi_A/\partial\gamma = 0$ at $\gamma = \frac{1+b}{1+2b}$ when $b = b_A^1$. The results follows from the concavity of $\Delta\pi_A$. □

**Proof of Proposition 5.3.9:**

*Proof.* The result follows from the concavity of $\Delta\pi_A$ and Proposition 5.3.7. □

**Proof of Proposition 5.3.10:**

*Proof.* The result follows from the fact that $\Delta\pi_A$ is increasing in $b$. □

**Proof of Proposition 5.3.11:**

*Proof.* Solve $\Delta\pi_A = \Delta\pi_{SC}$ for $\gamma$, we have the following four solutions:

$$\gamma = 0,\ 1,\ \frac{(1+b)\left(8-b-\sqrt{b^2+16}\right)}{8b} \equiv \gamma_1,\ and\ \frac{(1+b)\left(8-b+\sqrt{b^2+16}\right)}{8b} \equiv \gamma_2.$$

It is easy to check that $\gamma_2 > 1$. $\gamma_1 \geq 0$ if and only if $b \leq 3$. When $b > 3$, $\Delta\pi_A - \Delta\pi_{SC} \leq 0$ for any $0 < \gamma \leq \frac{1+b}{1+2b}$. When $\gamma_1 \geq \frac{1+b}{1+2b}$, we have

$\Delta\pi_A - \Delta\pi_{SC} \geq 0$ for any $0 < \gamma \leq \frac{1+b}{1+2b}$, which is equivalent to $0 < b \leq b_1$, where $b_1$ is the unique solution of $2b^3 + 3b^2 - 3b - 3 = 0$ in $(1, \infty)$. When $b_1 < b < 3$, we have $\Delta\pi_A - \Delta\pi_{SC} \geq 0$ when $0 < \gamma < \gamma_1$ and $\Delta\pi_A - \Delta\pi_{SC} \leq 0$ when $\gamma_1 < \gamma \leq \frac{1+b}{1+2b}$. We hence prove the results. $\square$

**The Six Regions in Table 5.5:**

The six regions are defined as follows:

$$
\begin{aligned}
R^1_{w,C} &= \left\{ \begin{array}{l} 0 \leq w_n \leq \frac{2(1-\gamma)}{2+\gamma} \textit{ and } 0 \leq w_p \leq \frac{\gamma w_n}{2},\ \textit{or} \\ \frac{2(1-\gamma)}{2+\gamma} \leq w_n \leq 1-\gamma \textit{ and } 0 \leq w_p \leq 1-\gamma-w_n \end{array} \right\}, \\
R^2_{w,C} &= \left\{ \begin{array}{l} \frac{2(1-\gamma)}{2+\gamma} \leq w_n \leq 1-\gamma \textit{ and } 1-\gamma-w_n \leq w_p \leq \frac{\gamma(3w_n-1+\gamma)}{4-\gamma},\ \textit{or} \\ 1-\gamma \leq w_n \leq \frac{2-\gamma}{2} \textit{ and } w_n-1+\gamma \leq w_p \leq \frac{\gamma(3w_n-1+\gamma)}{4-\gamma} \end{array} \right\}, \\
R^3_{w,C} &= \left\{ 0 \leq w_n \leq \frac{2(1-\gamma)}{2+\gamma} \textit{ and } \frac{\gamma w_n}{2} \leq w_p \leq \gamma \right\}, \\
R^4_{w,C} &= \left\{ \begin{array}{l} 1-\gamma \leq w_n \leq \frac{2-\gamma}{2} \textit{ and } 0 \leq w_p \leq w_n-1+\gamma,\ \textit{or} \\ \frac{2-\gamma}{2} \leq w_n \leq 1 \textit{ and } 0 \leq w_p \leq \frac{\gamma}{2} \end{array} \right\}, \\
R^5_{w,C} &= \left\{ \frac{2(1-\gamma)}{2+\gamma} \leq w_n \leq \frac{2-\gamma}{2} \textit{ and } \frac{\gamma(3w_n-1+\gamma)}{4-\gamma} \leq w_p \leq \gamma \right\}, \\
R^6_{w,C} &= \left\{ \frac{2-\gamma}{2} \leq w_n \leq 1 \textit{ and } \frac{\gamma}{2} \leq w_p \leq \gamma \right\}.
\end{aligned}
$$

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# Vita

Liwen Chen was born in Yiyang County of Hunan Province in P. R. China on 28 February 1974, the son of Dingzhong Chen and Bigeng Chen. Liwen received the Bachelor of Science degree in Applied Mathematics from Tsinghua University in 1996 and the Master of Science degree in Computer Science from National University of Singapore in 1999. After graduation, Liwen joined the Center for Natural Products Research as a Java Programmer. In 2000, Liwen moved to Abacus International as a Project Leader and Systems Analyst. In 2001, Liwen co-founded Beijing Lyang Technology Co. Ltd. in Beijing, P. R. China and acted as CEO and Chief Java Architect. After one year, Liwen founded Beijing WebNewVision Hi-Tech Limited and acted as CEO of the company. In 2004, Liwen entered Red McCombs School of Business at The University of Texas at Austin to pursue his PhD degree. Liwen earned his Master of Science degree in Operations Management in 2008. During his PhD study, Liwen participated in the UT-Chevron Operations Practicum teamed with two MBA students did a consulting project for ERCOT.

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